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Editors

# Localized Excitations in Nonlinear Complex Systems

Current State of the Art and Future  
Perspectives

 Springer

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# Preface

Localized excitations have been at the heart of developments of nonlinear dynamics (and especially of nonlinear complex systems) during the past few decades. Their names may vary (solitons, instantons, kinks, breathers, vortices, vortex rings, quodons, etc.), yet their relevance is ubiquitous and their importance undisputed not only in a wide variety of mathematical and theoretical developments but also in a broad range of applications. The latter constitute an ever-expanding list including, but certainly not limited to, Bose-Einstein condensates in atomic physics, optical fibers, waveguide arrays and photorefractive systems in nonlinear optics, Josephson junctions and arrays thereof, molecular crystals, layered silicates, ferromagnetic and antiferromagnetic materials, granular crystals, micromechanical cantilever or simple torsion pendulum arrays, electrical transmission lines, and nematic liquid crystals. Additionally, in many other cutting edge areas, they have been conjectured to not only emerge but also play a critical role in the dynamics and the explanation of experimental observations. Such fields include graphene nanoribbons, denaturation of the DNA double strand, atomic ejection and defect migration in crystals, and protein folding. It is thus clear that this is a subject which is fundamental for both theory and applications, and the evaluation of its state-of-the-art and the identification of the challenges that lie ahead are of paramount importance. Clearly, nonlinear science matures and continues to redefine itself, playing an increasingly important and ever-expanding role within the twenty-first century.

The 2nd Conference on “Localized Excitations in Nonlinear Complex Systems” (LENCOS’12) was organized with the aim of bringing together experimental and theoretical physicists, chemists, biologists, and applied mathematicians working in the broad field of nonlinear localized modes. This workshop was the second installment in the series, following up on the successful first conference that took place 3 years earlier (LENCOS’09). The latter meeting provided the opportunity for a special volume on the themes of the workshop which appeared in the journal *Discrete and Continuous Dynamical Systems – Series S (DCDS-S)*. Despite its short history, LENCOS has garnered considerable visibility and is already a widely appreciated venue for interdisciplinary exchanges and the promotion of research in this multifaceted and diverse thematic area. Both installments were

spearheaded by the Nonlinear Physics Group of the University of Sevilla, which had also organized some previous relevant workshops, namely, “Moving Breathers in Nonlinear Lattices” (2003), “Macroscopic Effects of Nonlinear Excitations” (2004), “New Horizons in Stochastic Complexity” (2004), and “Nonlinear Excitations: Theory and Experiments” (2005). The LENCOS’09 Conference was attended by approximately 65 delegates from Europe, America, and Asia, while this number grew to about 100 (and contained also delegates from Africa and Oceania) in 2012. Both meetings have featured a vibrant mix of senior and junior researchers, and numerous productive interactions have sprung thereafter as a result of the exchanges taking place in Sevilla.

The organizing committee of the 2nd LENCOS meeting felt that given the growth in scope and number of participants and of themes represented, it would be suitable on the occasion of the 2nd LENCOS meeting to assemble a wide range of state-of-the-art (nonlinear) science contributions in the present volume. In so doing, we also conscientiously decided to target a broader set of potential contributors than merely the conference delegates. Hence, we advertised the corresponding volume more broadly to attract an as rich and diverse set of topics and authors as possible. The resulting submissions were subjected to a full refereeing process, involving at least two referees for each paper. At this point, it is appropriate for us to take the opportunity to thank all referees for their timely, careful, and valuable work and contribution to the quality of the resulting volume. We also want to thank all authors both for submitting the relevant works and for taking care of all remarks by the referees and respecting the imposed timelines. We sincerely hope that the resulting outcome will be one that will be of value to the nonlinear waves and localized modes’ community, and will serve as a reference point for numerous future developments in this field.

In what follows, we give a brief overview of the topics that are represented in this volume, placing them in some rough, broad thematic categories accordingly.

**Nonlinear Schrödinger Models: Continuum and Discrete.** As may be natural to expect, the nonlinear Schrödinger equation (NLS) and its discrete counterpart (DNLS) have a considerable representation in the contents of this volume. The work of Achilleos, Kevrekidis, Frantzeskakis, and Carretero-González considers the context of solitary waves and vortices (and a novel type of excitation entitled a “ghost” state) in both one- and two-dimensional installments of the topic of nonlinear  $\mathcal{PT}$ -symmetric media; this theme has attracted considerable attention recently. On the other hand, Alfimov and Avramenko offer significant insights on the problem of complete description of nonlinear states in an NLS model with a periodic potential using encodings based on bi-infinite sequences of  $N$ -symbols. Additionally, Lepri and Casati examine the case of a nonlinear dimer embedded in an otherwise linear lattice, in a one-dimensional setting; they observe and explain the surprising feature of nonreciprocal wave propagation when wave packets scatter from the two different directions. Finally, James and Starosvetsky, motivated by a problem in granular crystals (the so-called cradle problem), examine a reduced equation, the so-called discrete- $p$ -Schrödinger equation (which is an interesting

variant of the DNLS); the authors find, through both map and quasi-continuum approaches, different analytical and numerical approximations to its solutions.

**Bose-Einstein Condensates (BECs) and Nonlinear Optics.** Many of the works in the volume concerned themselves with areas of application of nonlinear models, where localized excitations may arise. Among the most canonical, such areas are notably atomic physics and nonlinear optics. In the context of the former, the contribution of Balaž and Nicolin examines atomic condensates with attractive interatomic interactions under the influence of a periodic drive of the scattering length; the authors infer analytically and numerically that a resonant drive may lead the BEC into a fragmented state. On the other hand, Salasnich tackles the context of ultracold Fermi gases, examining the notion of the contact intensity, as well as using a hydrodynamic approach to explore their sound waves, response functions, and structure factors. Gerdjikov and Todorov consider the interaction of arrays of solitons (so-called  $N$ -soliton configurations) in a system of coupled NLS equations of the Manakov type; they consider, in particular, the influence of external potentials of the periodic, quadratic, and quartic type, with a view towards applications in BECs. In their work, in turn, Li, Pang, and Malomed study nonlinear localized modes trapped in a rotating ring with a modulated strength of the nonlinearity (both in a sign-definite and in a sign-changing case); the results are applicable both in BECs confined in a toroidal trap and in the propagation of light in a twisted pipe waveguide. Finally, Wang and Aceves touch upon propagation of light in two-dimensional plasmonic arrays by considering the specific setup of two-dimensional dielectric/metallic waveguides; the authors report a variety of results concerning localization, solitary wave formation, and dynamical instabilities.

**Nonlinear Dynamical Lattices.** Another subject that was broadly represented in the volume was that of lattice dynamical systems with a rather diverse array of associated contributions. In particular, Velarde, Ebeling, and Chetverikov consider localized excitations and anisotropic directional ordering in a two-dimensional lattice model of cuprate layers. Cisneros-Ake, Minzoni, and Velarde propose a variational approximation towards the study of electron trapping by soliton-like travelling excitations for one-dimensional lattices, emphasizing the case of an initially delocalized electron. Zolotaryuk presents a study of underdamped and strongly discrete sine-Gordon lattice equation in the presence of an oscillating force, considering the mode-locked kink states. Finally, Sánchez-Morcillo, Jiménez, Chaline, Bouakaz, and Dos Santos offer a tantalizing set of both theoretical and experimental observations suggesting deep connections between the spatiotemporal dynamics of a ring of coupled pendula (and their normal oscillation modes) with the dynamics of drops or bubbles.

**Quodons and Muscovite Mica.** A topic that can also be considered as a branch of nonlinear lattice dynamics, yet it received considerable attention in this volume in its own right and thus warrants a separate mention, concerns the study of quodons and related excitations in muscovite mica. In their contribution, Dubinko and Shapovalov develop a rate theory of the radiation-induced precipitation in

solids as modified by the non-equilibrium fluctuations from the gas of quodons produced by irradiation. On the other hand, Russell examines the long-range propagation of multiple-kink pulses in a crystal as a result of the energy gain from the crystal, bearing in mind as an application example the nuclear scattering of relativistic muons in muscovite crystals. Finally, Archilla, Kosevich, Jiménez, Sánchez-Morcillo, and García-Raffi present a model bearing supersonic kinks in Coulomb lattices. In their case, this is also intended as a prototypical model of layered silicates such as muscovite mica where the potassium ions form a two-dimensional lattice between silicate layers.

**Magnetic Metamaterials.** Two contributions in this volume addressed the subject of localized excitations that can emerge in magnetic metamaterials; hence we cluster them in the final coherent thematic entity presented herein. In particular, Diblík, Fečkan, Pospíšil, Rothos, and Susanto examine the potential formation of travelling waves in nonlinear magnetic metamaterials by considering a one-dimensional discrete array of nonlinear resonators. On the other hand, Pelinovsky and Rothos explore the discrete Klein-Gordon-type model originally derived by Eleftheriou, Lazarides, and Tsironis, obtaining a general criterion for the spectral stability of the multi-site discrete breathers in that model.

**Other Nonlinear Models.** Naturally, there were also a number of contributions that do not fit in a definitive way in any of the above categories. In this class, we mention the following: (i) the examination of small nonlinear stochastic arrays via cumulant expansions and numerical simulations of Morillo, Gómez-Ordóñez, and Casado; (b) the study of shape-invariant solitons in nonlocal models of nematic liquid crystals and the influence of noise in their breathing (under weak noise) and even their destruction (by strong noise) of Belić, Petrović, Strinić, and Aleksić; (c) the study of conservation laws for a family of reaction-diffusion-convection equations and the presentation of the notion of weak self-adjointness by Bruzón, Gandarias, and De la Rosa; and finally (d) the study of localized excitations in nonlinear complex biological systems as induced by high-density green photons in the contribution of Comorosan, Polosan, Apostol, Popescu, Farcanasu, Paslaru, and Ionescu.

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# Contents

## Part I Nonlinear Schrödinger Models: Continuum and Discrete

<b>Solitons and Their Ghosts in <math>\mathcal{PT}</math>-Symmetric Systems with Defocusing Nonlinearities</b> .....	3
V. Achilleos, P.G. Kevrekidis, D.J. Frantzeskakis, and R. Carretero-González	
1 Introduction .....	4
2 The Model and Some Analytical Insights .....	5
3 Ground State and Single Dark Soliton .....	8
3.1 Ground State .....	8
3.2 Single Dark Soliton .....	9
4 Multiple Dark Soliton States and Nonlinear $\mathcal{PT}$ Phase Transitions .....	13
4.1 Two-Dark-Soliton State .....	13
4.2 Three-Dark-Soliton State .....	17
4.3 Nonlinear $\mathcal{PT}$ Phase Transitions .....	19
4.4 The “Free Space” Case .....	21
5 Dark Soliton Dynamics .....	22
6 Ghost States and Their Dynamical Role .....	26
7 Symmetry Breaking and Nonlinear $\mathcal{PT}$ Phase Transitions in Two Dimensions .....	30
8 Conclusions .....	40
References .....	41
<b>Coding of Nonlinear States for NLS-Type Equations with Periodic Potential</b> .....	43
G.L. Alfimov and A.I. Avramenko	
1 Introduction .....	43
2 Bounded and Singular Solutions of Stationary States .....	45
2.1 Some Definitions .....	45
2.2 Some Statements About Collapsing Points .....	46

3	Periodic Potential: Poincaré Map .....	48
3.1	Poincaré Map .....	48
3.2	Symbolic Dynamics: Theory .....	49
3.3	Symbolic Dynamics: Application .....	51
4	Steady States with Cosine Potential .....	52
4.1	Hypothesis 1 .....	55
4.2	Hypothesis 2 .....	56
4.3	Hypothesis 3 .....	57
4.4	Examples .....	58
5	Conclusion .....	60
	References .....	60
	<b>Nonreciprocal Wave Propagation Through Open, Discrete Nonlinear Schrödinger Dimers</b> .....	63
	Stefano Lepri and Giulio Casati	
1	Introduction .....	63
2	The Model .....	65
3	Stability of Scattering Solutions .....	68
4	Scattering of Wave Packets .....	71
5	Summary and Conclusions .....	72
	References .....	74
	<b>Breather Solutions of the Discrete <math>p</math>-Schrödinger Equation</b> .....	77
	Guillaume James and Yuli Starosvetsky	
1	Introduction and Main Results .....	78
2	Two-Dimensional Mappings Equivalent to the Stationary DpS Equation .....	81
2.1	Force Variables .....	82
2.2	Mixed Variables .....	83
3	Stable and Unstable Manifolds .....	84
3.1	Heuristics .....	85
3.2	Stable Manifold Theorem .....	85
3.3	Stable and Unstable Manifolds for the Maps $M, T, F$ .....	87
4	Homoclinic Solutions of the Stationary DpS Equation .....	90
4.1	Homoclinic Solutions of Generalized DNLS Equations .....	90
4.2	Application to the DpS Equation .....	93
5	Analytical Approximations of Stable and Unstable Manifolds .....	96
5.1	Method of Local Approximation and Backward Iterates .....	97
5.2	Continuum Limit for Weak Nonlinearities .....	103
6	Breather Stability and Mobility .....	107
7	Discussion .....	112
	References .....	114

**Part II Bose-Einstein Condensates (BECs) and Nonlinear Optics**

**Fragmentation of a Bose-Einstein Condensate Through Periodic Modulation of the Scattering Length** ..... 119  
 Antun Balaž and Alexandru I. Nicolin

1 Introduction ..... 119

2 Classification of Density Waves ..... 121

3 Results and Discussion ..... 123

4 Conclusions ..... 127

References ..... 128

**Contact Intensity and Extended Hydrodynamics in the BCS-BEC Crossover** ..... 131  
 Luca Salasnich

1 Contact Intensity ..... 131

    1.1 Uniform Superfluid Fermi Gas at Zero Temperature ..... 133

    1.2 Contact Intensity in the BCS-BEC Crossover ..... 134

    1.3 Trapped Superfluid Fermi Gas ..... 135

2 Extended Superfluid Hydrodynamics ..... 137

    2.1 Extended Hydrodynamics in Terms of Goldstone Field ..... 140

    2.2 Application: The Unitary Fermi Gas ..... 141

    2.3 Nonlinear Sound Waves, Static Response Function and Structure Factor ..... 142

3 Conclusions ..... 145

References ..... 145

***N*-Soliton Interactions for the Manakov System: Effects of External Potentials** ..... 147  
 V.S. Gerdjikov and M.D. Todorov

1 Introduction ..... 147

2 Preliminaries ..... 150

    2.1 Derivation of the CTC as a Model for the Soliton Interaction of VNLS Systems ..... 150

3 Effects of the Polarization Vectors on the Soliton Interaction ..... 153

4 Derivation of the PCTC as a Model for the Soliton Interactions ..... 155

5 Comparison of the PCTC with the Perturbed Manakov Model ..... 158

    5.1 The Periodic Potentials ..... 158

    5.2 The Quadratic Potentials ..... 159

    5.3 The Quartic Potentials ..... 161

6 Discussion ..... 164

7 Conclusions ..... 165

References ..... 166

**Wave Modes Trapped in Rotating Nonlinear Potentials** ..... 171  
 Yongyao Li, Wei Pang, and Boris A. Malomed

1 Introduction ..... 172

2 The Model ..... 174

3 Numerical Results ..... 176

- 3.1 The Classification of Trapped Modes..... 176
- 3.2 The Self-Focusing Nonlinearity..... 177
- 3.3 The Self-Defocusing Nonlinearity ..... 181
- 3.4 The Alternating Self-Focusing: Self-Defocusing  
Nonlinear Potential ..... 183
- 4 The Analytical Approach ..... 184
  - 4.1 The Two-Mode Approximation ..... 184
  - 4.2 The Self-Focusing Nonlinearity..... 184
  - 4.3 The Self-Defocusing Nonlinearity ..... 185
  - 4.4 The Alternating Self-Focusing: Self-Defocusing Nonlinearity..... 187
- 5 Conclusions ..... 188
- References ..... 189

**Light Propagation in Two Dimensional Plasmonic Arrays..... 193**

Danhua Wang and Alejandro B. Aceves

- 1 Introduction ..... 193
- 2 Theoretical Model and Dispersion Properties: Case 1 ..... 194
- 3 Theoretical Model and Dispersion Properties: Case 2 ..... 198
- 4 Modulation Instability ..... 200
  - 4.1 Numerical Simulations ..... 203
- 5 Solitary Wave Solutions ..... 204
- 6 Conclusion ..... 207
- References ..... 208

**Part III Nonlinear Dynamical Lattices**

**Localized Excitations and Anisotropic Directional Ordering  
in a Two-Dimensional Morse Lattice Model of Cuprate Layers..... 213**

Manuel G. Velarde, Werner Ebeling, and Alexander P. Chetverikov

- 1 Introduction ..... 213
- 2 Model Hamiltonian and Visualization of Local and Collective  
Atom (Lattice Units) Motions ..... 216
- 3 Simulation of the Oxygen Dynamics ..... 220
- 4 Discussion and Conclusions..... 220
- References ..... 223

**Variational Approximation to Electron Trapping  
by Soliton-Like Localized Excitations in One-Dimensional  
Anharmonic Lattices..... 225**

Luis A. Cisneros-Ake, Antonmaria A. Minzoni,  
and Manuel G. Velarde

- 1 Introduction ..... 225
- 2 Formulation of the Problem and Numerical Results ..... 227
- 3 Traveling Polaron/Solelectron Solutions..... 232
- 4 Conclusions ..... 234
- References ..... 235

**Stability of Mode-Locked Kinks in the AC Driven and Damped Sine-Gordon Lattice** ..... 237  
 Yaroslav Zolotaryuk

- 1 Introduction ..... 237
- 2 The Model and Equations of Motion ..... 238
  - 2.1 The DSG Equation..... 238
  - 2.2 Linear Stability and the Floquet Theory ..... 239
- 3 Kinks in the High-Frequency Driven DSG Equation ..... 240
  - 3.1 The Numerical Scheme..... 241
  - 3.2 The Existence Diagram..... 241
  - 3.3 Floquet Spectrum and the Destabilizing Bifurcations ..... 244
- 4 Discussion and Conclusions ..... 249

References ..... 250

**Spatio-Temporal Dynamics in a Ring of Coupled Pendula: Analogy with Bubbles** ..... 251  
 V.J. Sánchez-Morcillo, N. Jiménez, J. Chaline, A. Bouakaz, and S. Dos Santos

- 1 Introduction ..... 252
- 2 The Ring of Coupled Pendula ..... 253
  - 2.1 The Model..... 253
  - 2.2 Experimental Setup ..... 254
- 3 Localized Excitations ..... 256
  - 3.1 Lower Cutoff Modes ..... 256
  - 3.2 Upper Cutoff Modes ..... 258
- 4 Defect-Induced Localization ..... 259
- 5 Analogy with Gas Bubbles in an Ultrasound Field..... 260
- 6 Conclusions ..... 261

References ..... 262

**Part IV Quodons and Muscovite Mica**

**Theory of a Quodon Gas with Application to Precipitation Kinetics in Solids Under Irradiation** ..... 265  
 V. Dubinko and R. Shapovalov

- 1 Introduction ..... 265
- 2 Gas of Quodons and Its Effect on Reaction Rates in Solids ..... 267
  - 2.1 Gas of Quodons ..... 268
  - 2.2 Modification of Reaction Rates in Solids Under Irradiation..... 270
- 3 Modeling of the Precipitation Kinetics Under Irradiation in the Modified Rate Theory ..... 274
  - 3.1 Classical Rate Theory of the Precipitation Kinetics ..... 275
  - 3.2 Quodon-Induced Solubility Limit Change ..... 279
  - 3.3 Quodon-Induced Interfacial Energy Change ..... 282
  - 3.4 Diffusivity of Cu Under Irradiation ..... 283

3.5 Evolution of Cu Precipitates and the Matrix Concentration of Cu Under Electron Irradiation .....	285
4 Summary .....	287
References .....	288
<b>Energy Gain by Discrete Particle Non-linear Lattice Excitations .....</b>	<b>289</b>
F.M. Russell	
1 Introduction .....	290
2 Experimental Methods .....	292
3 Layered Structure Effects .....	294
4 Molecular Dynamics Simulations of Cascades .....	296
5 Numerical Modelling in 2D Arrays .....	298
6 Structure of Non-linear Intrinsic Localised Modes .....	301
7 Energy Gain by Kink Pulses .....	304
8 Creation of Coordinated Kink Pulses .....	306
9 Evidence for Multiple Kink Pulses from Nuclear Scattering .....	307
10 Defect Sweeping .....	310
11 Inner Structure of Fans .....	310
12 Discussion .....	311
13 Conclusion .....	313
References .....	314
<b>Supersonic Kinks in Coulomb Lattices .....</b>	<b>317</b>
Juan F.R. Archilla, Yuriy A. Kosevich, Noé Jiménez, Víctor J. Sánchez-Morcillo, and Luís M. García-Raffi	
1 Introduction .....	318
2 Model Equations .....	319
3 Phonons .....	321
4 Localized Travelling Solutions .....	323
4.1 Tail Analysis .....	323
4.2 Oscillating Tails .....	324
4.3 Supersonic Solitons and Kinks .....	325
5 Kink Characteristics Within the Rotating Wave Approximation: Magic Wave Number .....	326
6 Numerical Results .....	328
7 Effect of the Interaction with Several Neighbors .....	330
8 Conclusions .....	330
References .....	331
<b>Part V Magnetic Metamaterials</b>	
<b>Travelling Waves in Nonlinear Magnetic Metamaterials .....</b>	<b>335</b>
J. Diblík, M. Fečkan, M. Pospíšil, V.M. Rothos, and H. Susanto	
1 Introduction .....	336
2 Existence Results on Periodic Solutions .....	337
3 Bifurcation Results for Periodic Travelling Waves .....	343

4 Bifurcation Results for Asymptotic Travelling Waves ..... 348

5 Numerical Results ..... 354

References ..... 357

**Stability of Discrete Breathers in Magnetic Metamaterials** ..... 359

Dmitry Pelinovsky and Vassilis Rothos

1 Introduction ..... 359

2 Formalism ..... 360

3 Existence of Multi-site Discrete Breathers ..... 362

4 Stability of Multi-site Breathers ..... 365

4.1 Adjacent Excited Oscillators ..... 366

4.2 Oscillators at Rest Between Excited Oscillators ..... 369

5 Discussion ..... 372

References ..... 374

**Part VI Other Nonlinear Models**

**Checking the Validity of Truncating the Cumulant Hierarchy**

**Description of a Small System** ..... 377

Manuel Morillo, José Gómez-Ordóñez, and José M. Casado

1 Introduction ..... 377

2 The Model ..... 379

3 Numerical Simulations ..... 380

4 Conclusion ..... 386

References ..... 387

**Shape-Invariant Solitons in Nematic Liquid Crystals:**

**The Influence of Noise** ..... 389

Milivoj R. Belić, Milan S. Petrović, Aleksandra I. Strinić,  
and Najdan B. Aleksić

1 Introduction ..... 389

1.1 Fundamental Solitons ..... 389

1.2 Nonlocality ..... 390

1.3 Status of the Field ..... 390

1.4 Contents of the Chapter ..... 391

2 The Model ..... 391

2.1 Scalar Nonlocal Model of Nematic Liquid Crystals ..... 391

2.2 Model Equations ..... 392

3 Computations ..... 393

3.1 Numerical Procedure ..... 393

3.2 Modified Petviashvili Procedure ..... 394

3.3 The Stability of Fundamental Solitons ..... 395

4 The Influence of Noise ..... 397

4.1 The Breathing Problem ..... 397

4.2 Propagation in the Presence of Noise ..... 397

4.3 Destructive Influence of Noise ..... 399

5	Conclusion .....	400
	References .....	400
<b>Conservation Laws of a Family of Reaction-Diffusion-Convection Equations .....</b>		<b>403</b>
M.S. Bruzón, M.L. Gandarias, and R. de la Rosa		
1	Introduction .....	403
2	Formal Lagrangian and Adjoint Equation .....	405
3	Lie Symmetries .....	406
4	Nonlinear Self-Adjoint Equations.....	409
5	Conservation Laws for a Subclass of Nonlinear Self-Adjoint Equations .....	411
6	Conclusions .....	416
	References .....	417
<b>Effects Induced in Complex Biological Systems by High Density Green Photons .....</b>		<b>419</b>
Sorin Comorosan, Silviu Polosan, Marian Apostol, Irinel Popescu, Ileana Farcasanu, Liliana Paslaru, and Elena Ionescu		
1	Introduction .....	420
2	Experimental Evidence .....	421
	2.1 Mass Spectroscopy .....	422
	2.2 Enzyme Reactions .....	424
3	Discusion and Conclusion: A Physical Model .....	426
	References .....	427
	<b>Index .....</b>	<b>429</b>



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