Preface

The phenomenon of Bose–Einstein condensation is a phase transition originally predicted by Bose and Einstein in 1924. In particular, it was shown that below a critical temperature T_c, a finite fraction of particles of a boson gas condenses into the same quantum state, known as the Bose–Einstein condensate (BEC). Although Bose–Einstein condensation was known to be a fundamental phenomenon, connected, e.g., to superfluidity and superconductivity, BECs were only experimentally realized 70 years later. This major achievement took place in 1995 and has already been recognized through the 2001 Nobel Prize in Physics. This first unambiguous manifestation of a macroscopic quantum state in a many-body system sparked an explosion of activity, as reflected by the publication of several thousand papers related to BECs since then. Nowadays there exist more than a hundred experimental BEC groups around the world, while an enormous amount of theoretical work has followed and driven the experimental efforts, with an impressive impact on many branches of physics.

From a theoretical standpoint, and for experimentally relevant conditions, the static and dynamical properties of a BEC can be described by means of an effective mean-field equation known as the Gross–Pitaevskii equation (GPE). This model is of the form

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta\Psi + g|\Psi|^2\Psi + V_{\text{ext}}(\mathbf{r})\Psi, \tag{1}$$

where $\Psi = \Psi(\mathbf{r}, t)$ is the BEC wavefunction (the atomic density is proportional to $|\Psi|^2$), Δ is the Laplacian, m is the atomic mass, and the prefactor g is proportional to the atomic scattering length. Notice that $g > 0$ (e.g., for ^{87}Rb and ^{23}Na atoms) or $g < 0$ (e.g., for ^{7}Li and ^{85}Rb atoms) corresponds, respectively, to repulsive or attractive interatomic interactions; accordingly, the nonlinearity in the GPE model is defocusing (repulsive) or focusing (attractive).

The GPE is a variant of the famous nonlinear Schrödinger (NLS) equation incorporating an external confining potential, V_{ext}. The NLS equation is known to be a universal model describing the evolution of complex field envelopes in nonlinear dispersive media. As such, the NLS equation is a key model appearing in a variety of physical contexts, ranging from optics to fluid dynamics and plasma physics, while it has also attracted much interest from a mathematical viewpoint. The relevance and importance of the NLS model is not limited to the case of conservative systems; in fact, the NLS equation is directly connected to dissipative universal models, such as the complex Ginzburg–Landau equation, which have been studied extensively in the context of pattern formation. In the case of BECs, the nonlinearity in the GPE (NLS) model is introduced by the interatomic interactions through an effective mean-field.

Importantly, the mean-field approach, and the study of the GPE, allows the prediction and description of important and experimentally relevant nonlinear effects and nonlinear waves, such as solitons and vortices. These so-called matter-wave solitons and vortices can be viewed as fundamental nonlinear excitations of BECs, and as such have attracted considerable attention. Importantly, they have also been observed in many elegant experiments using various relevant techniques. These include, among others, phase engineering in order to create vortices or dark matter-wave solitons in them, the stirring (or rotation) of the condensates creating vortices and vortex lattices, and the change of scattering length (from repulsive to attractive via Feshbach resonances) to produce bright matter-wave solitons and soliton trains. As far as vortices and vortex lattices are concerned, it should be noted that their description and connection to phenomena as rich and profound as superconductivity and superfluidity were one of the themes of 2003 Nobel Prize in Physics.

In an earlier project entitled "Emergent Nonlinear Phenomena in Bose–Einstein Condensates," we asked a number of experts to contribute their perspectives on a series of select topics at the interface of nonlinear science and atomic physics, within the exciting juncture of BECs. Yet, this was already 2007, and the level of developments in the field during the last 6–7 years has been remarkable. New results have enabled observations of nonlinear phenomena pertaining to the existence, evolution, and mutual interactions of dark solitons, vortices, and vortex rings. For instance, in quasi-1D geometries, studies of dark solitons and their collisions have emerged, and counterflow experiments have revealed spontaneous realizations of symbiotic solitons such as dark-bright and dark-dark ones. Quasi-2D experiments have showcased, through novel minimally destructive imaging techniques, the formation and direct observation of clusters containing a few vortices, and have offered a profound understanding of vortices, their motions induced by inhomogeneities of the background density, and their mutual interactions. Finally, the state-of-the-art experimental techniques are on the verge of being able to extract 3D in situ images that will provide long-standing feedback upon current frontiers, such as vortex lines and vortex rings, their intrinsic motions, and mutual interactions with direct implications to quantum turbulence, among many others.

While "The *Focusing* Nonlinear Schrödinger Equation" has enjoyed its share of attention through, among others, a volume on this very subject, we felt, in light of these recent developments, that "The *Defocusing* Nonlinear Schrödinger Equation" has received considerably less attention in the form of a coherent volume that would collect many of these recent results. The aim of the present book is to bridge this gap, by presenting a wide array of findings on the realm of BECs and on the NLS-type models that arise therein.

In addition to an overarching introduction to the relevant class of models, the book contains a chapter focused on 1D and quasi-1D settings, which is centered around dark-soliton excitations. There is also a chapter on 2D and quasi-2D case examples which, in turn, chiefly evolves around vortices and their dynamics. Finally, we also give a 3D formulation of the relevant system, with a particular aim towards understanding the dynamics of vortex lines and vortex rings. While the presentation is naturally rather biased towards numerous themes of our particular interest and recent work, we hope that the collection of the relevant background, theoretical developments, numerical computations, and experimental results will be of rather wide interest to atomic physicists, nonlinear scientists, and applied mathematicians. Furthermore, we hope that this book will be equally valuable to beginners on the field, as well as to experienced researchers in the area.

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February 2015

Acknowledgments

Most of the material included in this manuscript would not have come to fruition without the invaluable contribution of our collaborators (including many of our past and present research students). We are very grateful to all of our collaborators for the time they have lent us towards the fascinating study of nonlinear waves and especially nonlinear Schrödinger-type problems. Thanking each individual would be unwieldy (there are several hundred); however, it is clear in our minds that the research journey we have taken over the last 15 years would not have been possible without the continuous exchange of ideas with all the collaborators, co-authors, and colleagues with whom we have had the pleasure to interact, as well as our students and postdocs due to their timeless effort and energy invested in this program.

We would also like to acknowledge the financial support that has been generously given to us towards our research efforts. In particular we would like to thank the following organizations for their support over the last decade or so: the National Science Foundation (NSF), The University of Athens, the Computational Science Research Center (CSRC) at San Diego State University (SDSU), the Los Alamos National Laboratory (LANL), the Department of Energy (DoE), the Binational Science Foundation, the Alexander von Humboldt Foundation, Germany, the European Research Council (ERC), and the US Air Force Office of Scientific Research (US-AFOSR).