1. In the lectures, we had a solution to the logistic growth model, so we could readily perform a nonlinear least squares fit of the data to the solution. Suppose that the differential equation describing the population dynamics did not have an explicit solution, so had to be solved numerically. Create a scheme (program block diagram) that describes how you would find the nonlinear least squares fit to the parameters in this differential equation, which must be solved numerically, to the experimental data. Provide sufficient details that you could readily write such a program in MatLab or some similar programming language (which already has nice packages for solving differential equations). Note the features that you would want as far as passing variables.

2. Consider the data on the yeast that is posted on our webpage (yeast.dat). This problem has the reader analyze the yeast data and fit a logistic growth model using different techniques.

   a) Using the first \((t = 0)\) and fifth point \((t = 4)\) estimate the growth rate for this population. Use the first point \((t = 0)\) as \(N_0\) and the last point \((t = 18)\) as the carrying capacity. Write the explicit fit obtained by using these parameters.

   b) Use the first 5 data points \((t = 0, 1, 2, 3, 4)\) to find the best fit of an exponential function to the data using a nonlinear square routine (cf. `lsqcurvefit` in MatLab). Use these values for initial population \(N_0\) and growth rate \(R_0\), together with the estimation of the carrying capacity found in a) to write an explicit fit to the data.

   c) Repeat b) by using the logarithm of the first 5 data points \((t = 0, 1, 2, 3, 4)\) and fitting a line. Write an explicit fit to the data using the carrying capacity estimated in a).

   d) Perform a nonlinear square fit (using preferably `lsqcurvefit` in MatLab) of the whole data set using the exact solution for the logistic growth. Write the estimation for the initial population \(N_0\), the growth rate \(R_0\) and the carrying capacity. Write the explicit fit obtained by using these parameters.

   e) Plot, in the same graph, the original data and the 4 fitted models a)-d). Give a brief discussion of how well they model the growing culture.

3. Consider

\[
\begin{align*}
\frac{dx}{dt} &= ax + by \\
\frac{dy}{dt} &= cx + dy
\end{align*}
\]

for the following cases:

(a) \(a = 1, \quad b = 3, \quad c = 1, \quad d = -1\)

(b) \(a = 4, \quad b = -3, \quad c = 1, \quad d = 0\)

(c) \(a = -1, \quad b = 2, \quad c = -2, \quad d = -1\)

A) Find the solution for \((x(t), y(t))\) using eigenvalues/eigenvectors.

B) Determine the stability of the origin.

C) Sketch the phase plane with isoclines and eigenvectors (you may want to use `pplane6` for this).

4. Ex. 46.4, p 203 (Haberman)