NONLINEAR BEHAVIOUR IN THE JBD SCALAR-TENSOR THEORY

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ABSTRACT

We apply techniques of nonlinear dynamics to a cosmological problem in the Jordan Brans-Dicke theory. The solutions presented here show irregular oscillatory behaviour in the scale factors and a positive Liapunov exponent in the Bianchi IX model. This is evidence of stochastic behaviour in the model.

1. Introduction

Modern gravitation theories are highly nonlinear, for example Einstein equations

$$R_{ab} - \frac{1}{2}Rg_{ab} = 8\pi G T_{ab}, \tag{1}$$

provide 10 coupled nonlinear partial differential equations in four variables to determine 10 metric functions. This nonlinear characteristic has proven to be one of the main obstacles for the proper understanding of the possible range of dynamical behaviour in a nonsymmetric spacetime.

There has been attempts of applying nonlinear dynamics (NLD) techniques to problems of cosmological interest[1-3], and even some interest in uncovering its chaotic properties. Zardecki[4], Rugh and Jones,[5] and others have investigated the irregular behaviour of cosmological Bianchi models in general relativity (GR). The most conspicuous result of these investigations is the ongoing discussion about the

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meaning of NLD in gravitational theories. On the other hand, to our knowledge there has been no attempt of investigating the possible stochastic behaviour in alternative theories of gravitation.

2. Scalar–Tensor theory

We report here results obtained so far on the possible stochastic behaviour in the scalar–tensor cosmological theory of Jordan Brans–Dicke (JBD), this theory is perhaps the most serious contender of (GR). JBD includes Mach principle (GR does not) introducing an additional scalar field, ϕ, responsible of the "inertia producing" effects of distant matter.

The specific model we work on is spatially homogeneous, with a line element given by

$$ds^2 = -dt^2 + g_{\mu\nu}(t)\omega^\mu\omega^\nu, \quad \mu,\nu = 1,2,3, \tag{2}$$

where ω^b are 1-forms in the orbits of the group and

$$g_{\mu\nu} = \ \mathrm{diag}(a_1^2(t), a_2^2(t), a_3^2(t)). \tag{3}$$

this metric is a function only of synchronous time t and the $a_i(t)$ with (i=1, 2, 3) are the scale factors for the model. The JBD field equations reduce to a system of nonlinear ordinary diferential equations (ODE's) for the a_i-s and the scalar field ϕ. In the specific case of the Bianchi IX vacuum model the equations are

$$\begin{aligned}\left(\frac{\ddot{a}_i}{a_i}\right) - \left(\frac{\dot{a}_i}{a_i}\right)^2 + \left(\frac{[a_1a_2a_3]^{\dot{}}}{a_1a_2a_3} + \frac{\dot{\phi}}{\phi}\right)\left(\frac{\dot{a}_i}{a_i}\right) + \left(\frac{1}{a_i^2}\right) + \\ \frac{1}{2}\left(\frac{a_i^2}{a_j^2a_k^2} - \frac{a_j^2}{a_k^2a_i^2} - \frac{a_k^2}{a_i^2a_j^2}\right) = 0,\end{aligned} \qquad i = 1,2,3. \tag{4}$$

for the scalar field, we have an extra equation:

$$\frac{d}{dt}[\dot{\phi}a_1a_2a_3] = 0. \tag{5}$$

These equations have proven to be difficult to solve[6,7], but they admit the first integral

$$\begin{aligned}C := \left(\frac{\dot{a}_1\dot{a}_2}{a_1a_2}\right) + \left(\frac{\dot{a}_1\dot{a}_3}{a_1a_3}\right) + \left(\frac{\dot{a}_2\dot{a}_3}{a_2a_3}\right) + \left(\frac{[a_1a_2a_3]^{\dot{}}}{a_1a_2a_3}\right)\left(\frac{\dot{\phi}}{\phi}\right) - \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \\ \frac{1}{2}\left(\frac{1}{a_1^2} + \frac{1}{a_2^2} + \frac{1}{a_3^2}\right) - \frac{1}{4}\left(\frac{a_1^2}{a_2^2a_3^2} + \frac{a_2^2}{a_3^2a_1^2} + \frac{a_3^2}{a_1^2a_2^2}\right) = 0,\end{aligned} \tag{6}$$

the equation above may be considered as a constriction on the specific model to be satisfied by the initial conditions.

Given a set of initial conditions ($a_i(0)$, $\phi(0)$, $\dot{a}_i(0)$, $\dot{\phi}(0)$) we have obtained numerically the evolution of the scale factors (figure 1) and the scalar field (figure 2) in this model.

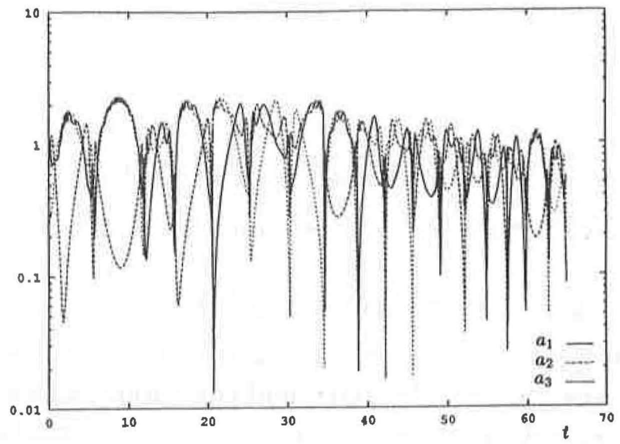

Figure 1. Time evolution of the scale factors a_1, a_2, a_3, plotted in a logarithmic scale.

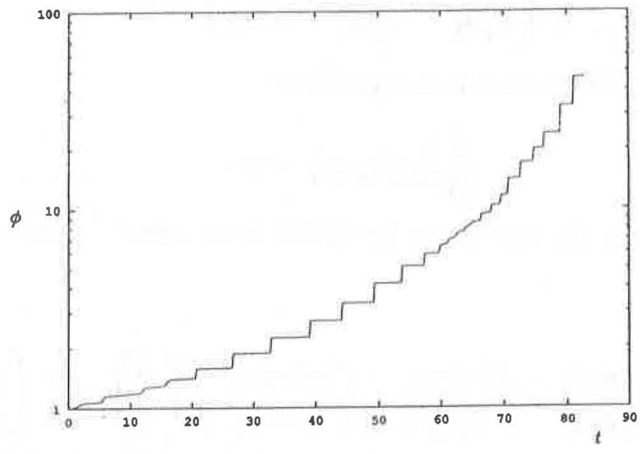

Figure 2. Logarithmic time evolution of the scalar field ϕ.

For the sake of completeness, a 3D-plot of the scale factors (a_1 vs a_2 vs a_3) is also shown in figure 3. There are several papers about Bianchi IX model in GR[4,5,8] which have concluded stochastic behaviour in this model. We show that this kind of behaviour appears too in JBD theory in the "same" cosmological model. A result that might be very important for cosmological considerations.

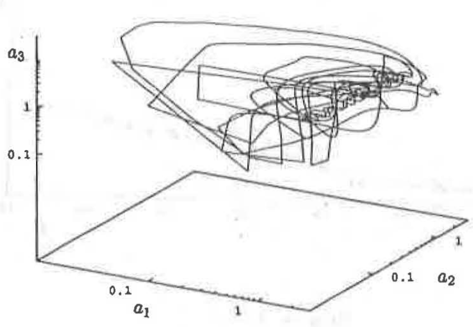

Figure 3. Logarithmic evolution of the three scale factors.

There are several ways of characterising chaos in a system. In this paper we evaluate the maximal Lyapunov exponent,[9] λ, for the model using the relation

$$\lambda_t = \frac{1}{2t}\log\frac{d_t(\xi)}{d_{t_0}(\xi)}, \tag{7}$$

where

$$d_t(\xi) = \sum_{i=1}^{3}(\delta a_i)^2 + (\delta\phi)^2 + \sum_{i=1}^{3}(\delta\dot{a}_i)^2 + (\delta\dot{\phi})^2. \tag{8}$$

The Lyapunov exponent (LE) λ is defined as the limit $t \to \infty$ of expression (7)^{1}0. Figure 4 shows the numerical results we have got for the LE as a plot $\lambda_t$vs$t$. The tendency is clear, λ is positive. This evidence has led us to guess stochastic properties for the model.

We have analysed numerically the properties of the JBD Bianchi IX model. The results we have got may be taken as evidence that the model exhibits stochastic behaviour. The calculations were made using a fourth-order Runge- Kutta routine from the CERN computer library and our own program for evaluating the maximum Liapunov exponent.

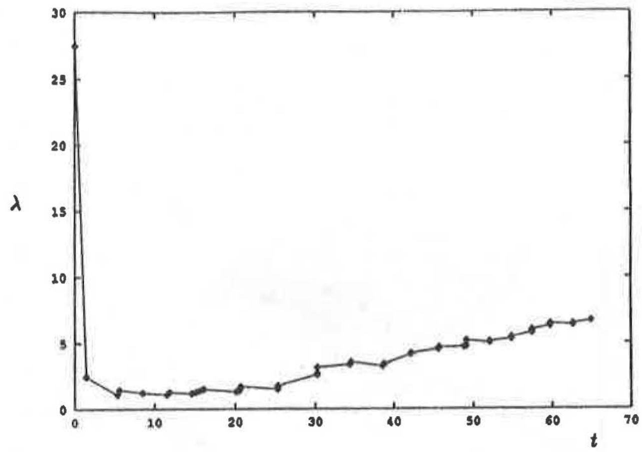

Figure 4.
Succesive estimates of the maximum Lyapunov λ_t exponent plotted against the synchronous time t.

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