Calculus I Review for Final Exam

Chapter 1.1: Four Ways to Represent a Function

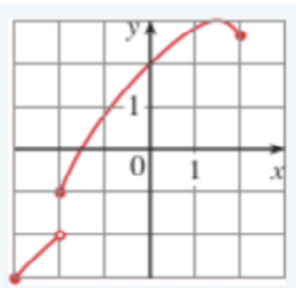
Q1) $f(x) = \dfrac{x^2 - 4}{x - 2}$

(a) Find the domain of $f(x)$.

(b) Find the range of $f(x)$.

(c) Sketch a graph of $f(x)$.

Q2) Does the graph below represent a function? If so, what are the domain and range?



Chapter 1.2: Mathematical Models: A Catalogue of Essential Functions

Q3) What do all members of the family of linear functions $f(x) = 3x + c$ have in common?

Q4) Find an expression for a cubic function $f(x)$ where $f(1) = 6$ and $f(-1) = f(0) = f(2) = 0$.

Q5) The relationship between the Fahrenheit (F) and Celsius (C) temperature scales is given by $F = \frac{9}{5}C + 32$.

(a) Sketch a graph of this function.

(b) What is the slope of the graph and what does it represent? What is the F-intercept and what does it represent?

Chapter 1.3: New Functions from Old Functions

Q6) Graph the following functions using transformations (i.e. without plotting points) by starting with the graph of a known function.

(a) $y = -x^2$

(b) $y = 2\cos(3x)$

(c) $y = 1 - \dfrac{1}{x}$

Chapter 1.4: Exponential Functions

Q7) Find the domain of the following functions:

(a) $f(x) = \dfrac{A+B+C}{1-e^{1-x^2}}$

(b) $f(x) = \dfrac{940 + Ax^4}{e^{\cos(x)}}$

Q8) Use exponent laws to simplify $\dfrac{x^3 \cdot x^n}{x^{n+1}}$.

Chapter 1.5: Inverse Functions and Logarithms

Q9) Given that $f(x) = \ln(x+3)$, find the inverse function $f^{-1}(x)$.

Q10) A population of cats begins with 100 cats and doubles every 3 years. Let $N(t)$ be the number of cats after t years.

(a) Find the function $N(t)$.

(b) Find the inverse of the function $N(t)$ and explain its meaning (e.g. $N(t)$ represents the number of cats after t years).

(c) Use the inverse to find how many years it takes for the population of cats to reach 50000.

Q11) Find the domain and range of $y = \arcsin(3x+1)$.

Chapter 2.1: The Tangent and Velocity Problems

Q12) A tank holds 1000 gallons of water, which drains from the bottom of the tank in half an hour. The values in the table below show the volume V of water remaining in the tank (in gallons) after t minutes.

t (min)	5	10	15	20	25	30
V (gal)	694	444	250	111	28	0

(a) If P is the point $(15, 250)$ on the graph of V, find the slopes of the secant lines PQ when Q is the point on the graph for each of $t = 5, 10, 20, 25$, and 30.

(b) Estimate the slope of the tangent line at P by averaging the slopes of two secant lines

Chapter 2.2: The Limit of a Function

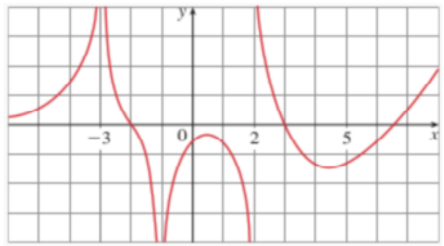
Q13) Find the following limits.

(a) $\displaystyle\lim_{x\to 3^-} \frac{\sqrt{x}}{(x-3)^5}$

(b) $\displaystyle\lim_{x\to \frac{\pi}{2}^+} \frac{1}{x}\sec(x)$

(c) $\displaystyle\lim_{x\to 2\pi^-} x\csc(x)$

Q14) Consider the graph of $f(x)$ below.



(a) $\displaystyle\lim_{x\to -3} f(x) =$

(b) $\displaystyle\lim_{x\to -1} f(x) =$

(c) $\displaystyle\lim_{x\to 2^-} f(x) =$

(d) $\displaystyle\lim_{x\to 2^+} f(x) =$

(e) $\displaystyle\lim_{x\to 2} f(x) =$

Chapter 2.3: Calculating Limits Using the Limit Laws

Q15) Given that

$$\lim_{x\to 2} f(x) = 4, \qquad \lim_{x\to 2} g(x) = -2, \qquad \lim_{x\to 2} h(x) = 0,$$

find the following limits if they exist. If they do not exist, explain why.

(a) $\lim_{x\to 2}[f(x) + 5g(x)]$

(b) $\lim_{x\to 2} \sqrt{f(x)}$

(c) $\lim_{x\to 2} \frac{g(x)}{h(x)}$

(d) $\lim_{x\to 2} \frac{3f(x)}{g(x)}$

(e) $\lim_{x\to 2} \frac{g(x)h(x)}{f(x)}$

Chapter 2.5: Continuity

Q16) Use the definition of continuity and the properties of limits to determine if each of the following functions is continuous at the given number a.

(a) $f(x) = (x + 2x^3)^4$ and $a = -1$

(b) $p(v) = 2\sqrt{3v^2 + 1}$ with $a = 1$

Q17) Use the Intermediate Value Theorem to show that there is a root of the given equation in the specified interval (a, b).

(a) $x^4 + x - 3 = 0$ on $(1, 2)$

(b) $e^x = 3 - 2x$ on $(0, 1)$

Chapter 2.6: Limits at Infinity

Q18) Find the following limits.

(a) $\lim_{x\to\infty} \frac{4x+3}{5x-1}$

(b) $\lim_{x\to\infty} e^{-2x}\cos(x)$

(c) $\lim_{x\to-\infty} \frac{\sqrt{1+4x^6}}{2-x^3}$

(d) $\lim_{x\to\infty} (\sqrt{x^2+ax} - \sqrt{x^2+bx})$

Chapter 2.8: The Derivative as a Function

Q19) Find the derivative of each of the following functions using the definition of derivative. State the domain of the function and the domain of its derivative.

(a) $f(x) = 3x - 8$

(b) $g(t) = \frac{1-2t}{3+t}$

(b) $g(t) = \sqrt{9-x}$

Chapter 3.1: Derivatives of Polynomials and Exponential Functions

Q20) Find the derivative of each of the following functions:

(a) $g(x) = \dfrac{1}{\sqrt{x}} + \sqrt[4]{x}$

(b) $f(x) = 3e^x + \dfrac{4}{\sqrt[3]{x}}$

(c) $g(x) = 4\pi x^2$

(d) $h(x) = \dfrac{\sqrt{x} + x}{x^2}$

(e) $f(z) = \dfrac{A + Bz + Cz^2}{z^2}$

Chapter 3.2: The Product and Quotient Rules

Q21) Find $f''(1)$ if $f(x) = \dfrac{x^2}{1+x}$.

Q22) Find $f'(x)$ and $f''(x)$ for each of the following functions.

(a) $f(x) = \dfrac{x}{x^2 - 1}$

(b) $f(x) = e^x \sqrt{x}$

Chapter 3.3: Derivatives of Trigonometric Functions

Q23) Find the derivative of each of the following functions.

(a) $f(x) = \cos^2(x)$

(b) $f(x) = e^x \sin(x) + \cos(x)$

(c) $f(x) = \dfrac{\cos(x)}{1 - \sin(x)}$

Q24) Find the equation of the tangent line to the curve at the given point

(a) $f(x) = \sin(x) + \cos(x)$ at the point $(0, 1)$

(b) $f(x) = x + \sin(x)$ at the point (π, π)

Chapter 3.4: The Chain Rule

Q25) Find the derivative of each of the following functions.

(a) $G(x) = e^{C/x}$

(b) $F(t) = \tan\left(\sqrt{1 + t^2}\right)$

(c) $f(t) = e^{at} \sin(bt)$

Chapter 3.5: Implicit Differentiation

Q26) Find $\frac{dp}{ds}$ if $3p^2 - \cos(p) = s^3$.

Q27) Find the equation of the tangent line that passes through the point $(1, 2)$ on the graph of $8y^3 + x^2y - x = 3$.

Chapter 3.6: Derivatives of Logarithmic Functions

Q28) Find the derivative $\frac{d}{d\theta}$ of $f(\theta) = \ln(A\sin(\theta) + e^{\theta} + B)$.

Q29) Find the derivative $\frac{d}{ds}$ of $f(s) = \ln(\ln(g(s)))$.

Chapter 3.8: Exponential Growth and Decay

Q30) Given that strontium has a half-life of 28 days, answer the following.

(a) A sample has an initial mass of 50 mg. Find a formula $m(t)$ for the mass remaining after t days.

(b) Find the mass remaining after 40 days.

(c) After how long will only 1 mg remain?

Q31) A sample of tritium-3 decayed to 94.5% of its original amount after one year.

(a) What is the half-life of tritium-3?

(b) How long would it take for the sample to decay to 20% of its original amount?

(c) If a sample of tritium-3 begins with a mass of 327 mg, after how many years will only 42 mg remain?

Chapter 3.9: Related Rates

Q32) The height of a triangle is increasing at a rate of 1 cm/min, while the area of the triangle is increasing at a rate of 2 cm^2/min. At what rate is the base of the triangle changing when the height is 10 cm and the area is 100 cm^2?

Q33) At noon, ship A is 100 km west of ship B. Ship A is sailing south at 35 km/hr and ship B is sailing north at 25 km/hr. How fast is the distance between the ships changing at 4:00 pm?

Chapter 3.10 Linear Approximations and Differentials

Q34) Find the linearization $\mathcal{L}(x)$ of the function $f(x) = x^3 - x^2 + 3$ at $a = -2$.

Q35) Find the linear approximation $\mathcal{L}(x)$ of the function $g(x) = \sqrt[3]{1+x}$ at $a = 0$, then use $\mathcal{L}(x)$ to approximate the values $\sqrt[3]{0.95}$ and $\sqrt[3]{27.05}$.

Chapter 4.1: Maximum and Minimum Values

Q36) Find the absolute maximum and absolute minimum values of $f(x) = x - 2\tan^{-1}(x)$ on the interval $[0, 4]$. Use the fact that $\dfrac{d}{dx}\tan^{-1}(x) = \dfrac{1}{1+x^2}$.

Q37) Find the maximum value of $f(x) = x^a(1-x)^b$ on $0 \le x \le 1$, where a and b are positive constants.

Chapter 4.2: The Mean Value Theorem

Q38) Verify that the function $f(x) = \ln(x)$ satisfies the conditions of the Mean Value Theorem on the interval $[1, 4]$, then find all values c such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

Q39) Suppose that $4 \le f'(x) \le 10$ for $3 \le x \le 9$. Show that $24 \le f(9) - f(3) \le 60$.

Chapter 4.4: Indeterminate Forms and L'Hospital's rule

Q40) Find the following limits:

(a) $\displaystyle\lim_{x\to 1} \frac{\sqrt{x-1}}{x-1}$

(b) $\displaystyle\lim_{x\to\infty} \left(\frac{8}{x}\right)^{x}$

(c) $\displaystyle\lim_{x\to\infty} x^3 e^{-x}$

Chapter 4.7: Optimization Problems

Q41) The profit for a company is modeled by $P(x) = x(150 - x) - 1600$, where x is the number of items sold each day. Find the number of items that the company needs to sell to achieve a maximum profit, then find the maximum profit.

Q42) A wire of length l is cut into two parts and one of the pieces is bent into the shape of a square while the other is bent into the shape of a triangle. Let ℓ_t be the length of the wire that is bent into a triangle and let ℓ_s be the length of the wire that is bent into a square. What are the lengths of the two pieces of wire that minimize the sum of the areas of the two shapes?

Chapter 4.9: Antiderivatives

Q43) Find each of the following general antiderivatives.

(a) $f(x) = A\sec^2(Bx) + E\cot(x)\csc(x) + De^{Bx}$

(b) $f(x) = a\sec(x)(\sec(x) + \tan(x)) + x^{e+A} - \dfrac{1}{Bx} + ax^n$

(c) $f(x) = \dfrac{A}{Bx} + E\cos(Bx) + D\csc(ax)(\csc(ax) - \cot(ax))$

Chapter 5.1: Areas and Distances

Q44) A ball is thrown vertically toward the ground from the top of a tall building with an initial velocity of 100 ft/sec. Its velocity after t seconds is given by $v(t) = 32t + 100$. How far does the ball fall between 1 and 3 seconds?

(a) How far does the ball fall between 1 and 3 seconds?

(b) Does the height of the building matter? Why or why not?

(c) Sketch a graph (be sure to include units) to show how you can graphically estimate the distance traveled between 1 and 3 seconds.

Chapter 5.2: The Definite Integral

Q45) Evaluate the following definite integrals.

(a) $\int_{-1}^{2}(4x^2 + x + 2)dx$

(b) $\int_{0}^{3}(5x + 2)dx$

Q46) Calculate the integral $\int_{0}^{1} |2x - 1|dx$ as an area.

Chapter 5.3: The Fundamental Theorem of Calculus

Q47) Use the Fundamental Theorem of Calculus (Part 1) to find the derivative of each of the following.

(a) $f(x) = \int_{0}^{x} t^2 dt$

(b) $f(x) = \int_{1}^{x^2+1}(e^t + t)dt$

(b) $f(x) = \int_{x}^{\cos(3x)+3} t^2 dt$

Q48) Suppose $a(x) = \int_{f(x)}^{g(x)} b(u)du$ and $a'(x) = \dfrac{\cos(\ln(3x^3))}{x} - 9x^2\cos(3x^2)$.

(a) What is the function $b(u)$?

(b) Find $g'(x)$ and $f'(x)$.

Chapter 5.4: Indefinite Integrals and the Total Change Theorem

Q49) Suppose that Peter is driving at a constant 50 miles per hour.

(a) Find a function $v(t)$ that represents Peter's speed at time t.

(b) How many miles has Peter traveled after 5 hours?

(c) Find the integral of $v(t)$ from 0 to 5.

(d) How does the Total Change Theorem apply here?

Q50) Suppose that Peter is inside a spacecraft that accelerates at a constant 50 mi/hr^2 from rest (i.e. $v(t) = 0$).

(a) Find a function $v(t)$ that represents Peter's speed at time t.

(b) How far has Peter traveled after 5 hours?

(b) How far does Peter travel between hour 5 and hour 7?

Chapter 5.5: The Substitution Rule

Q51) Evaluate the following integrals.

(a) $\int_2^3 \frac{x}{1+x^2} dx$

(b) $\int_0^1 xe^{-x^2} dx$

(c) $\int \frac{\sec(x)}{(\sec(x)+\tan(x))^2} dx$

(d) $\int \frac{1}{x\ln(x)} dx$

(e) $\int (a \sin(abx) + 3a^2x)dx$

(f) $\int_p^q Ax^{1/2}(C + Bx^{3/2})dx$

(g) $\int \frac{\ln(3x^2+5))^2}{3x^2+5}dx$