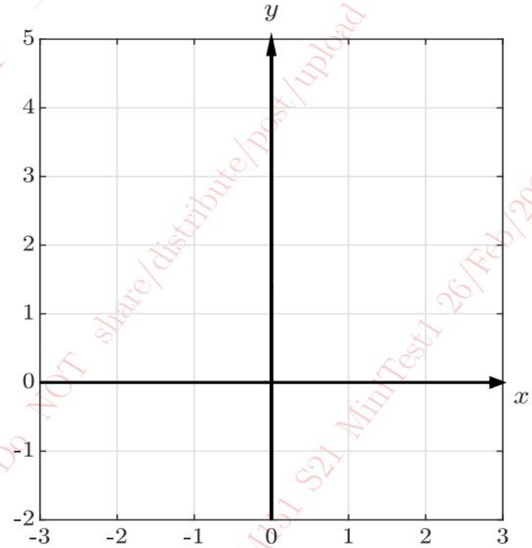
Sec. 6.1: Areas Between Curves

2. **(3 pts)** [SDSU M151 S21 MiniTest1 V1 Q02 26/Feb/2021 5:00-6:10pm PDT — Do NOT share/distribute/post/upload]
[Remember to **stop** solving and **submit** when there are **10 mins** (or more) left on the clock!!! No late submissions!]

Find an integral (or integrals) for the area enclosed by the curves:

$$y = f(x) = x^2 + 2x \quad \text{and} \quad y = g(x) = -x^2 + 4.$$

You do **NOT** need to compute the integral(s).
Sketch the curves!!!



$A =$

Sec. 6.2: Volumes (by slices, disks and washers)

4. **(4 pts)** [SDSU M151 S21 MiniTest1 V1 Q04 26/Feb/2021 5:00-6:10pm PDT — Do NOT share/distribute/post/upload]
[Remember to **stop** solving and **submit** when there are **10 mins** (or more) left on the clock!!! No late submissions!]

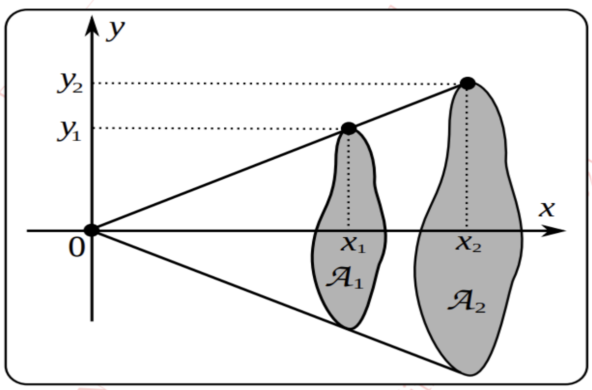
Volumes by slices.

The law of similar triangles states that

$$\frac{x_2}{x_1} = \frac{y_2}{y_1},$$

with the distances as depicted in the figure. Similarly, it can be shown that the areas depicted in the figure are related through

$$\frac{\mathcal{A}_2}{\mathcal{A}_1} = \left(\frac{x_2}{x_1}\right)^2 = \left(\frac{y_2}{y_1}\right)^2.$$

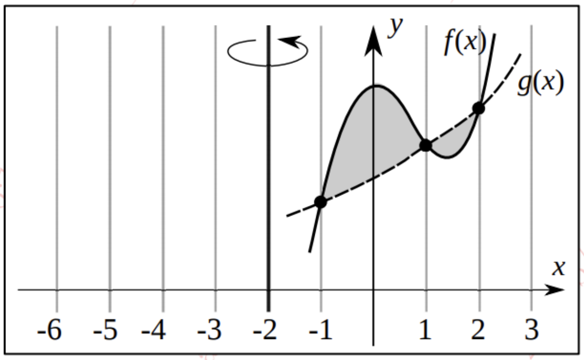


Use this fact and the methods learned in class to compute the volume of a cone of height H and with an **arbitrary shape** base of area $\mathcal{A}$. Draw a diagram clearly indicating all objects/labels that you used.

Sec. 6.3: Volumes by Cylindrical Shells

3. **(4 pts)** [SDSU M151 S21 MiniTest1 V1 Q03 26/Feb/2021 5:00-6:10pm PDT — Do NOT share/distribute/post/upload] [Remember to **stop** solving and **submit** when there are **10 mins** (or more) left on the clock!!! No late submissions!]

Using the method of volumes by **SHELLS**, write an integral (or integrals) for the solid generated by rotating about the $\boldsymbol{x = -2}$ **line** (note that the line is off-axis!) the shaded region on the figure. Do NOT forget to sketch a typical shell for this object.



$V =$

Sec. 7.1: Integration by Parts

b) **(3 pts)** $I_2 = \int x^a \ln(x)\, dx$, where a is a constant with $a > 1$.

$I_2 =$

Sec. 7.2: Trigonometric Integrals

a) **(3 pts)** $I_1 = \int \cos^3(x) \sin^q(x)\, dx$, where q an even constant.

$I_1 =$

Sec. 7.3: Trigonometric Substitution

6. **(4 pts)** [`SDSU M151 S21 MiniTest1 V1 Q06 26/Feb/2021 5:00-6:10pm PDT` — Do NOT share/distribute/post/upload]
[Remember to **stop** solving and **submit** when there are **10 mins** (or more) left on the clock!!! No late submissions!]

Using **trigonometric substitution** REWRITE the following x-integral as a θ-integral containing only trigonometric functions. Do NOT compute the integral!

$$I_4 = \int_0^3 \frac{x^3}{\sqrt{x^2+9}}\, dx.$$

$$I_4 = \int_{\square}^{\square} \boxed{} \, d\theta$$

Sec. 7.4: Integration of Rational Functions by Partial Fractions

4. **(4 pts) Partial Fractions:**
[`SDSU M151 F20 MiniTest2 V1 Q4 02/Oct/2020 3:00-4:10pm PDT` — Do NOT share/distribute/post/upload]
[Remember to **stop** solving and **submit** when there are **10 mins** (or more) left on the clock!!! No late submissions!]

Integrate $I_4 = \dfrac{7x+5}{x^2+x-2}$

$I_4 =$

Sec. 7.8: Improper Integrals

5. **(3 pts) Improper integrals:**
[SDSU M151 S21 MiniTest2 V2 Q5 19/Mar/2021 5:00-6:10pm PDT — Do NOT share/distribute/post/upload]
[Remember to **stop** solving and **submit** when there are **10 mins** (or more) left on the clock!!! No late submissions!]

Let $\mathbf{0 \leq h(x) \leq 3x}$ for $\mathbf{x \geq 0}$. Using the Comparison Theorem,**(a)** determine whether the integral

$$\boldsymbol{I_2 = \int_1^{\infty} \frac{2 + h(x)}{x^3}\, dx},$$

converges or diverges. **(b)** If convergent give an **upper bound**. Show all work!

(a) Convergence for I_2 :

(b) Upper bound for I_2 :

Sec. 8.1: Arc Length

2. **(4 pts) Arclength:** Write BOTH an $\boldsymbol{x}$ AND a $\boldsymbol{y}$ integral for the length of the curve

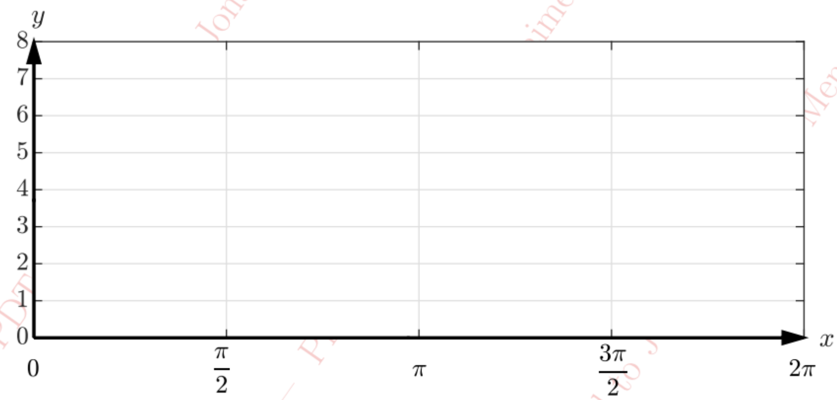
$\boldsymbol{y = f(x) = 4 + 3\cos x}$ for $\boldsymbol{0 \leq x \leq 2\pi}$.

Graph the function. You do NOT need to compute the integral! Hint#1: you may need one of these derivatives:

- $[\arccos x]' = [\cos^{-1} x]' = -\dfrac{1}{\sqrt{1-x^2}}$
- $[\arcsin x]' = [\sin^{-1} x]' = \dfrac{1}{\sqrt{1-x^2}}$

Hint#2: for the y integral you might want to use the symmetry of the function.

[SDSU M151 S21 MiniTest2 V2 Q2 19/Mar/2021 5:00-6:10pm PDT — Do NOT share/distribute/post/upload]
[Remember to **stop** solving and **submit** when there are **10 mins** (or more) left on the clock!!! No late submissions!]



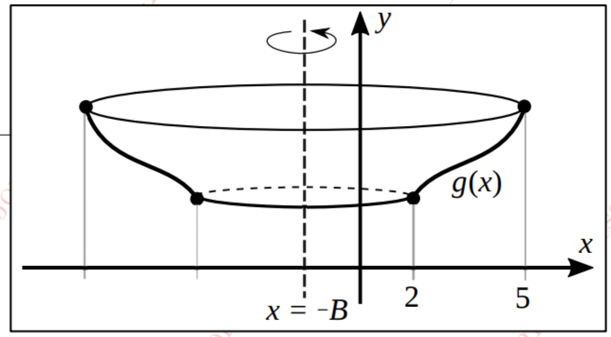
Over x: $L_x = \int_{\square}^{\square} \square \, dx$

Over y: $L_y = \int_{\square}^{\square} \square \, dy$

Sec. 8.2: Area of a Surface of Revolution

3. **(4 pts) Areas for surfaces of revolution:**
[SDSU M151 S21 MiniTest2 V2 Q3 19/Mar/2021 5:00-6:10pm PDT — Do NOT share/distribute/post/upload]
[Remember to **stop** solving and **submit** when there are **10 mins** (or more) left on the clock!!! No late submissions!]

Write BOTH an x and a y integral for the SURFACE AREA obtained, by rotating about the line $\boldsymbol{x = -B}$, the function $g(x)$ as depicted in the plot.
Note that rotation is NOT about the y-axis!



$S_x = \int_{\square}^{\square} \boxed{}\, dx$

$S_y = \int_{\square}^{\square} \boxed{}\, dy$

Sec. 9.3: Separable Equations

6. **(4 pts) Differential Equations:**
[SDSU M151 S21 MiniTest2 V2 Q6 19/Mar/2021 5:00-6:10pm PDT — Do NOT share/distribute/post/upload]
[Remember to **stop** solving and **submit** when there are **10 mins** (or more) left on the clock!!! No late submissions!]

For the following differential equation: $\boldsymbol{2y' - \frac{1}{y} = 0}$

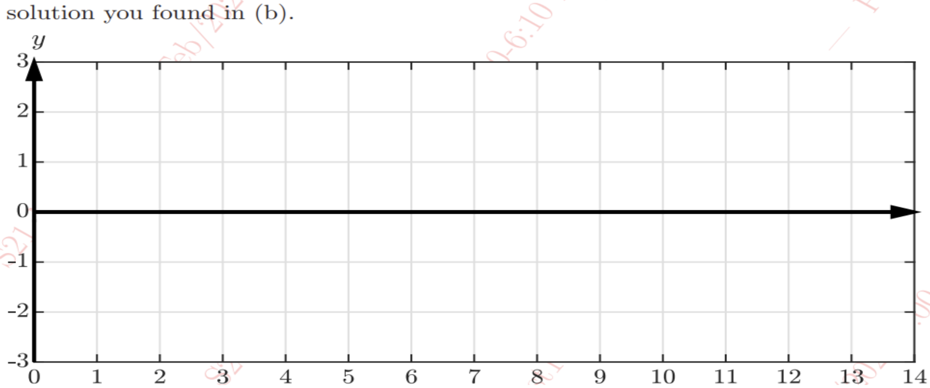
(a) Find the general solution

(a) General sol: $y(x) =$

(b) Find the particular solution satisfying the initial condition: $\boldsymbol{y(5) = -1}$.

(b) Particular sol: $y(x) =$

(c) Sketch a few solutions (encompassing the WHOLE available space in the graph below) using THIN lines and sketch using a THICK line the particular solution you found in (b).



Sec. 9.5: Linear Equations

2. **(3 pts) Linear Differential Equations:**
[`SDSU M151 S21 MiniTest3 V1 Q2 9/Apr/2021 5:00-6:10pm PDT` — Do NOT share/distribute/post/upload]
[Remember to **stop** solving and **submit** when there are **10 mins** (or more) left on the clock!!! No late submissions!]

(a) Give first GENERAL solution and then **(b)** the PARTICULAR solution satisfying the initial condition.

$$\frac{dy}{dx} - A\,y - 2\,e^{Ax} = 0 \text{ with } y(0) = 5, \quad \text{where } A \text{ is a constant.}$$

(a) General solution: $y(x) =$

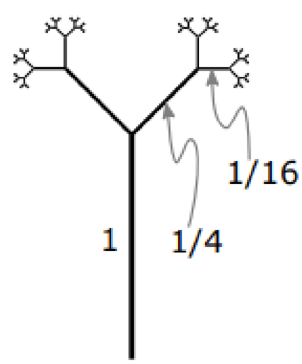
(b) Particular solution: $y(x) =$

[extra (+1 pts)]: Verify that the **general** solution you found does indeed solve the differential equation:

Sec. 11.2: Series (only geometric series and its applications)

9. **(5 pts)** Many plants and animals have developed roots and vascular systems that optimize the intake/exchange of environmental resources. This has lead to many of these system to take fractal shapes. Assume we have a branching system where each mother branch splits into **TWO** daughter branches and so on as depicted in the figure.

Assume in our case that the main mother branch has a length $\ell_0 = 1$ and that all daughter branches have a length ℓ_{i+1} that is $1/4$ of their mother branch length ℓ_i [i.e. $\ell_{i+1} = \ell_i/4$]. Compute the TOTAL LENGTH L of this branch system (including ALL branches) after an infinite number of splits. [Note that there is only one mother branch!]



Sec. 11.6: Ratio Test: interval and radius of convergence (NO absolute convergence and NO Root test)

7. **(3 pts)** Using the **RATIO test**, determine the **radius** AND the **interval** of convergence of the following infinite series. **Do NOT study convergence at the end points.** Explain what you are doing and show all your work!
[`SDSU M151 S21 MiniTest3 V1 Q9 9/Apr/2021 5:00-6:10pm PDT` — Do NOT share/distribute/post/upload]
[Remember to **stop** solving and **submit** when there are **10 mins** (or more) left on the clock!!! No late submissions!]

$$S_4 = \sum_{n=0}^{\infty} (-1)^n \frac{n^2}{5^n} (2x-3)^n$$

Radius of conv.:	Interval of conv.: $< x <$

Sec. 11.9: Representations of Functions as Power Series

Use the fact that the exponential function $e^x = \sum_{n=0}^{\infty} x^n$, find the series representation (using the Σ notation) for the following functions:

(a) $g(x) = \frac{d}{dx}\left[4\,x\,e^{x^3}\right]$

$g(x) = \sum_{n=0}^{\infty}$ [ ]

(b) $h(x) = \int e^{x^2}\,dx$ [The constant C is already written for you].

$h(x) = \sum_{n=0}^{\infty}$ [ ] $+ C$

Sec. 11.10: Taylor and Maclaurin Series (NO Taylor inequality and NO remainder)

Compute the Taylor polynomial of order 2 (i.e. second degree polynomial) for $\boldsymbol{f(x) = A\cos(\omega x + \phi)}$ about $\boldsymbol{x = B}$, where $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{\omega}$, $\boldsymbol{\phi}$, are CONSTANTS.

$f(x) \approx$

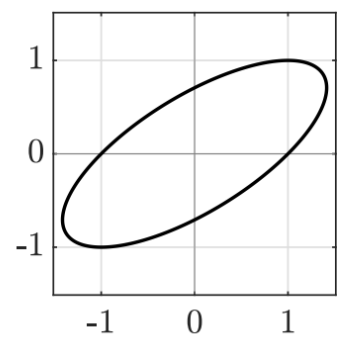
Sec. 10.1: Curves Defined by Parametric Equations
Sec. 10.2: Calculus with parametric Equations

Consider the following parametric equation for $\boldsymbol{0 \le t \le 2\pi}$:

$$\begin{cases} \boldsymbol{x(t) = \sin(t) + \cos(t)} \\ \boldsymbol{y(t) = \sin(t)} \end{cases}$$

whose graph is depicted in the figure.
[Hint: $\cos(\pi/4) = \sin(\pi/4) = \sqrt{2}/2$]

a) Using calculus, find ALL the points (x, y) that correspond to vertical AND horizontal tangency points. Show ALL your work... No work shown $\rightarrow$ no points!



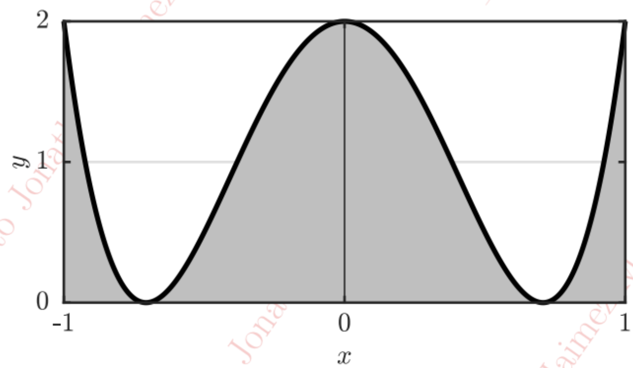
Horizontal	$t =$, $(x,y)=\left(\quad , \quad \right)$	$t =$, $(x,y)=\left(\quad , \quad \right)$
Vertical:	$t =$, $(x,y)=\left(\quad , \quad \right)$	$t =$, $(x,y)=\left(\quad , \quad \right)$

Consider the following parametric curve:

$$\begin{cases} x(t) = \sin(t) \\ y(t) = 1 + \cos(4t) \end{cases} \quad \text{for } -\frac{\pi}{2} \le t \le \frac{\pi}{2},$$

whose graph is depicted in the figure.

a) Write an explicit integral for the TOTAL length of the curve. [You do NOT need to compute the integral].



$L = \int_{\square}^{\square} \square \, d\square$

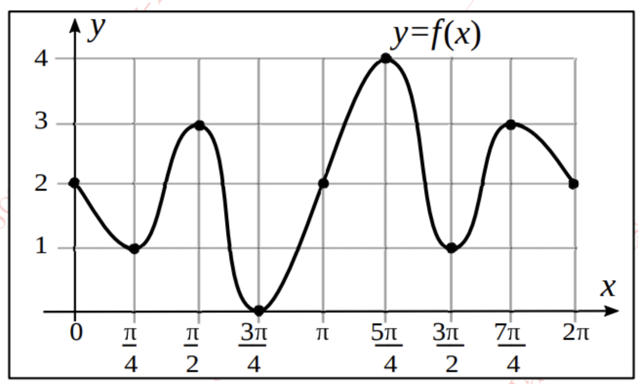
b) Write an explicit integral for the shaded area. [You do NOT need to compute the integral].

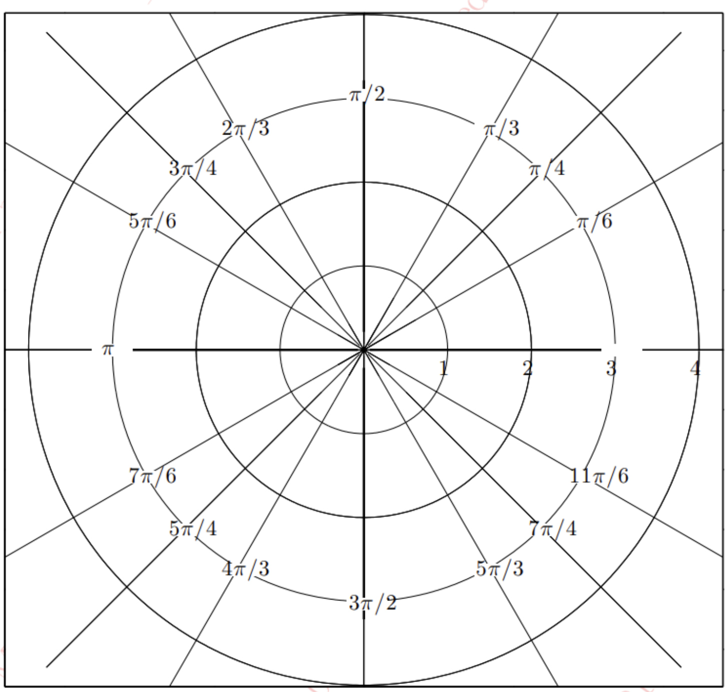
$A = \int_{\square}^{\square} \square \, d\square$

Sec. 10.3: Polar Coordinates

7. **(2.5 pts) Polar Equations:**
[`SDSU M151 S21 MiniTest4 V2 Q7 30/Apr/2021 5:00-6:10pm PDT` — Do NOT share/distribute/post/upload]
[Remember to **stop** solving and **submit** when there are **10 mins** (or more) left on the clock!!! No late submissions!]

The figure to the right represents the Cartesian $[(x, y)]$ plot of $y = f(x)$. Use this graph to sketch the **polar** curve $[(r, \theta)]$ traced by $r = f(\theta)$ for $0 \le \theta \le 2\pi$.





Sec. 10.4: Areas and Lengths in Polar Coordinates

$$A = \int_a^b \tfrac{1}{2} r^2 \, d\theta \qquad\qquad L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \, d\theta$$

17–21 Find the area of the region enclosed by one loop of the curve.

20. $r = 2 \sin 5\theta$

45–48 Find the exact length of the polar curve.

45. $r = 2\cos\theta, \quad 0 \leqslant \theta \leqslant \pi$