Sample problems to prepare for MT#2 –Math 151–Calculus II–Spring 2020

1. Integrals.

a) Integrate $I = \displaystyle\int \frac{5x+1}{x^2+x-2}\,dx =$

b) Integrate $I = \displaystyle\int \frac{2x^3+3x^2-12x-35}{x^2-x-6}\,dx =$

c) Write the partial fraction decomposition for the following integral.
JUSTIFY each term! You do NOT need to compute the coefficients:

$$I = \int \frac{3x^4-2x^2-5x+7}{(x^2+1)\,(x^2+4)^2\,(x-1)^2\,(x+1)\,(8x-3)}\,dx$$

d) Integrate $I = \displaystyle\int \frac{1}{\sqrt{x^2+8}}\,dx =$

e) Integrate $I = \displaystyle\int \frac{x^2}{\sqrt{4-x^2}}\,dx =$

2. Determine whether or not the following improper integrals converge or diverge (please explain!).
Compute, if possible, the value of the integral in the case of convergence.

a) $I = \displaystyle\int_3^\infty \frac{2}{3\,x^2}\,dx.$

b) $I = \displaystyle\int_2^4 \frac{2}{x-3}\,dx.$

c) $I = \displaystyle\int_\pi^\infty \frac{\cos^2(x)}{x^2}\,dx.$

d) $I = \displaystyle\int_1^\infty \frac{1}{3x^2+e^{2x}}\,dx.$

e) $I = \displaystyle\int_1^\infty \frac{2}{x}+\frac{3}{x^2}\,dx.$

3. Solve the following differential equations satisfying the given initial conditions. (i) Give first the general solution and then (ii) the particular solution solving for the initial condition.
Note: always remember to check first if the differential equation is SEPARABLE!

a) $\dfrac{dy}{dx} - 2\,x\,y = 2\,e^{x^2}$ with $y(0)=1$.

b) $y'\,y - x\,\cos(x) = 0$ with $y(0)=0$.

c) $x\,y^2\,y' = x-3$ with $y(1)=1$.

d) $x\,y' - 4y = x^5\,e^x$ with $y(1)=2$.

4. Write an x-integral AND a y-integral giving the length of the curve defined by the graph of $y = f(x) = \ln(x)$ for $1 \leq x \leq 4$. You do NOT need to compute these integrals. Draw a sketch **including the coordinates of the initial and final points**!

5. Write an x-integral AND a y-integral giving surface of revolution generated by rotating about the x-axis the curve $y = \sqrt{x}$ for $1 \leq x \leq 4$. You do NOT need to compute these integrals. Draw a sketch **including the coordinates of the initial and final points**!

6. Write an x-integral AND a y-integral giving surface of revolution generated by rotating about $y = 3$ (note this is off-axis!) the curve $y = f(x)$ for $\pi \leq x \leq 5$. Consider that the graph of the curve is always ABOVE the line $y = 3$. You do NOT need to compute these integrals. Draw a sketch **including the coordinates of the initial and final points**!

7. Show, using the methods learned in class, that the surface area of the cone of circular base of radius r and height h is $A = \pi r \sqrt{h^2 + r^2}$ (do NOT include the area of the base). DRAW A SKETCH!!! Clearly indicate which method you are using, the function(s) that you are plotting, and the interval of integration.

8. Show, using the methods learned in class, that the surface area of sphere of radius R is $A = 4\,\pi\,R^2$. DRAW A SKETCH!!! Clearly indicate which method you are using, the function(s) that you are plotting, and the interval of integration.

9. A population $P(t)$ behaves according to the differential equation: $\dfrac{dP}{dt} = f(P) = (P-1)\,(P-3)\,(P-5)\,(P-8)$. Perform the following tasks:

(a) Draw a sketch for $f(P)$ as a function of P. [You do not need to tabulate the function! Just use the roots to draw a rough sketch!] Find (and indicate in the sketch) the roots of f.

(b) Give the population points and/or intervals where the population is (i) constant, (ii) increasing, and (iii) decreasing.

(c) Sketch (i) the slope field and (ii) all qualitatively different solutions (i.e., all constant solutions and samples of all increasing/decreasing ones).