Math 151
Spring 2019
Apr 2018

Name(Print) ______________________
RedID ______________________
Section #/TA ___________

MIDTERM 3 PRACTICE PROBLEMS

*You do **not** need to turn this worksheet in.*

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1. Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues. Be sure to specify where "n" begins.

$$\left\{\frac{1}{2}, -\frac{4}{3}, \frac{9}{4}, -\frac{16}{5}, \frac{25}{6}, \ldots\right\}$$

2. Determine whether the geometric series is convergent or divergent. If it is convergent, find its sum.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n^3} + \frac{5^n}{3^n}\right)$

3. Determine whether the series is convergent or divergent by expressing s_n as a telescoping sum. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \frac{3}{n(n+1)}$$

4. Express the number as a ratio of integers. $1.\bar{8} = 1.8888\ldots$

5. A doctor prescribes a 100-mg antibiotic tablet to be taken every eight hours. Just before each tablet is taken, 20% of the drug remains in the body.

   (a) How much of the drug is in the body just after the second tablet is taken? After the third tablet?

   (b) If Q_n is the quantity of the antibiotic in the body just after the nth tablet is taken, find an equation that expresses Q_{n+1} in terms of Q_n.

   (c) What quantity of the antibiotic remains in the body in the long run?

6. Use the Integral Test to determine whether the series is convergent or divergent.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

(b) $\displaystyle\sum_{n=1}^{\infty} n^2 e^{-n^3}$

7. Determine whether the series converges or diverges. (Direct Comparison Test)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3+8}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{6^n}{5^n-1}$

8. Determine whether the series converges or diverges. (Limit Comparison Test)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{1+n}}{2+n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^4+1}}{n^3+n}$

9. Test the series for convergence or divergence. (Alternating Series Test)

(a) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3+5n}$ (b) $\sum_{n=2}^{\infty} (-1)^{n+1} \frac{n^2}{n^3+4}$

10. Test the series for convergence or divergence. (Test for divergence)

(a) $\sum_{n=1}^{\infty} \frac{e^n}{n^2}$ (b) $\sum_{n=1}^{\infty} (-1)^n \frac{n^2-1}{n^2+1}$

11. Determine whether the series is absolutely convergent or conditionally convergent.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$ (b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3+1}$

12. Find the radius of convergence and interval of convergence of the series.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n^2}$ (b) $\sum_{n=1}^{\infty} \frac{x^n}{n!}$ (c) $\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2+1}$

13. Express the function as the sum of a power series.

(a) Use differentiation to find a power series representation for $f(x) = \frac{1}{(1+x)^2}$
What is the radius of convergence?

(b) Use part (a) to find a power series for $g(x) = \frac{1}{(1+x)^3}$

(c) Use part (b) to find a power series for $h(x) = \frac{x^2}{(1+x)^3}$

14. Show that the function

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

is a solution of the differential equation

$$f''(x) + f(x) = 0.$$

15. Use the definition of a Taylor series to find the first four nonzero terms of the series for $f(x)$ centered at the given value of a.

(a) $f(x) = xe^x, \quad a = 0$

(b) $f(x) = \sin x, \quad a = \pi/6$

(c) $f(x) = \sqrt{x}, \quad a = 4$

(d) $f(x) = \ln(1+x), \quad a = 0$