

MIDTERM 3 PRACTICE PROBLEMS - Answers

1. $a_n = \frac{(-1)^{n+1}n^2}{n+1}$, starting with $n = 1$

2. (a) $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n}$ converges to $\frac{1}{7}$

(b) $\sum_{n=1}^{\infty} \left(\frac{1}{n^3} + \frac{5^n}{3^n} \right)$ is divergent

3. $\sum_{n=1}^{\infty} \frac{3}{n(n+1)}$ converges to 3

4. $1.\bar{8} = 1.8888\dots = \frac{17}{9}$

5. (a) After second tablet: 120 mg
After third tablet: 124 mg

(b) $Q_{n+1} = 0.2 Q_n + 100$.

(c) After n th tablet: $Q_n = \sum_{i=1}^n 100(0.2)^{i-1}$

In the long run: $\sum_{i=1}^{\infty} 100(0.2)^{i-1} = 125$

6. (a) $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ is divergent

(b) $\sum_{n=1}^{\infty} n^2 e^{-n^3}$ is convergent

7. (a) $\sum_{n=1}^{\infty} \frac{1}{n^3+8}$ converges

(b) $\sum_{n=1}^{\infty} \frac{6^n}{5^n-1}$ diverges

8. (a) $\sum_{n=1}^{\infty} \frac{\sqrt{1+n}}{2+n}$ is divergent

(b) $\sum_{n=1}^{\infty} \frac{\sqrt{n^4+1}}{n^3+n}$ is divergent

9. (a) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3+5n}$ is convergent
 (b) $\sum_{n=2}^{\infty} (-1)^{n+1} \frac{n^2}{n^3+4}$ is convergent
10. (a) $\sum_{n=1}^{\infty} \frac{e^n}{n^2}$ diverges
 (b) $\sum_{n=1}^{\infty} (-1)^n \frac{n^2-1}{n^2+1}$ diverges
11. (a) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$ is conditionally convergent
 (b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3+1}$ is absolutely convergent

12. (Note: question didn't ask to check endpoints)

- (a) $R = 1, I = (-1, 1)$
 (b) $R = \infty, I = (-\infty, \infty)$
 (c) $R = 1, I = (1, 3)$

13. Express the function as the sum of a power series.

- (a) $\sum_{n=1}^{\infty} (-1)^{n+1} n x^{n-1}, R = 1$
 (b) $\sum_{n=2}^{\infty} \frac{(-1)^n n(n-1) x^{n-2}}{2}$
 (c) $\sum_{n=2}^{\infty} \frac{(-1)^n n(n-1) x^n}{2}$

14.

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$f''(x) = \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n-2}}{(2n-2)!} = \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{2n}}{(2n)!}$$

Then plug in to $f''(x) + f(x) = 0$.

15. (a) $f(x) \approx x + x^2 + \frac{x^3}{2} + \frac{x^4}{6}$
 (b) $f(x) \approx \frac{1}{2} + \frac{\sqrt{3}}{2} \left(x - \frac{\pi}{6}\right) - \frac{1}{4} \left(x - \frac{\pi}{6}\right)^2 - \frac{\sqrt{3}}{12} \left(x - \frac{\pi}{6}\right)^3$
 (c) $f(x) \approx 2 + \frac{1}{4} (x-4) - \frac{1}{64} (x-4)^2 + \frac{1}{512} (x-4)^3$
 (d) $f(x) \approx x - \frac{1}{2} x^2 + \frac{1}{3} x^3 - \frac{1}{4} x^4$