Must Know Material for Mini-test#2 - M151 - Calculus II - Spring 2021

This sheet contains a list of the material that MUST be second nature to you in preparation for Mini-test#2. In addition to studying the following Calc-II material that will be included in Mini-test#2:

- Sec. 7.8: Improper Integrals
- Sec. 8.1: Arc Length
- Sec. 8.2: Area of a Surface of Revolution
- Sec. 9.1: Modeling with Differential Equations
- Sec. 9.2: Direction Fields ~~and Euler's Method~~
- Sec. 9.3: Separable Equations (inc. orthogonal curves AND mixing problems)
- Sec. 9.4: Models of Population Growth [Please refer to activity#8: link]

You must also be very confident with ALL the material from Calc-I. You can have a look at the following review material from Calc-I:

- [Derivatives] [Practice problems with solutions]
- [Integrals] [Practice problems with solutions]

You must also be very confident with ALL the material covered in Mini-tests #1 and #2. You can have a look at the following "Must know material for previous Mini-Tests":

- `http://carretero.sdsu.edu/teaching/M-151/MTs/MiniTest1_must_know.pdf`

In addition to studying ALL Calc-I and Calc-II material above, you must be very confident with the following basic and fundamental topics/formulas/techniques/tricks/hints/etc.:

- ALWAYS use the limit $t \to \pm\infty$ and $t \to a^{\pm}$ for, respectively, improper integrals of type I and type II.
- Comparison (sandwich) Theorem for improper integrals. Do not forget to check if all functions are ≥ 0.
- Arc length formula: $L = \int_a^b ds$ were $ds = \sqrt{1 + (dy/dx)^2}dx$ or $ds = \sqrt{1 + (dx/dy)^2}dy$.
- Area of revolution: $A = \int_a^b \text{circ} \times ds = \int_a^b 2\pi r \times ds = \int_{x_1}^{x_2} 2\pi f(x)\sqrt{1 + (f'(x))^2}dx$ or $\int_{y_1}^{y_2} 2\pi g(y)\sqrt{1 + (g'(y))^2}dy$.
- Solve DEs by separation of variables: do not forget constant (that will be determined by initial conditions). Note: always try separation of variables before integrating factor.
- Knowing the difference between the general solutions to the DE (the one with the arbitrary constant) and the particular solution (the one that passes through a specific initial condition).
- Being able to determine when a population is increasing/decreasing/stationary from the differential equation: $dP/dt = f(P)$. Namely, where is the function f positive/negative/zero respectively.
- Drawing the 2D direction field for $y' = f(x, y)$ (inc. trajectories starting at particular initial conditions).
- Being able to quickly sketch (i.e. without tabulating) all basic functions: lines, parabolas (including roots and vertex), logarithms, exponentials, trigonometric, and polynomials (using only their roots and their limits at $\pm\infty$).
- Remember that when solving something like: $y^2 = bla \Rightarrow y = \pm\sqrt{bla}$ or $|y| = bla \Rightarrow y = \pm(bla)$ you should not forget the $\pm$, otherwise you throwing away HALF of the solutions!
- When simplifying constants in the general solution to DEs remember that $3C_1 \to C_2$ or $C_1 + C_2 \to C_3$ where C_1, C_2, C_3 are arbitrary constants. However keep in mind that if C is an arbitrary constant: if $e^C \to K$ then $K \leq 0$ or if $\cos(C) \to K$ then $-1 \leq K \leq 1$, etc...
- If the problem has a constant (like α, A, β, etc.) just carry it through! It is a constant for the problem and it should be left untouched. Think about the way that you carry π around in expressions without using its actual numerical value [in fact it is *much* easier to write π than writing 3.141592653589793238462...].

Also, be careful with algebra! See must-know material for previous Mini-Test and also:

- $e^{a\ \ln(x)}$ is NOT $a\,x$ but $e^{a\ \ln(x)} = e^{\ln(x^a)} = x^a$.