Must Know Material for Mini-test#3 - M151 - Calculus II - Spring 2021

This sheet contains a list of the material that MUST be second nature to you in preparation for Mini-test#3. In addition to studying the following Calc-II material that will be included in Mini-test#2:

- Sec. 9.5: Linear Equations (Integrating Factor)
- Sec. 11.1: Sequences
- Sec. 11.2: Series (inc. Geometric Series and Telescopic)
- Sec. 11.3: Integral Test ~~and Estimates of Sums~~ (inc. p-Test)
- Sec. 11.4: The Comparison Tests (Sandwich/Direct Comparison Test and Limit Comparison Test)
- Sec. 11.5: Alternating Series
- Sec. 11.6: Absolute Convergence and the **Ratio Test** ~~and Root Test~~
- Sec. 11.7: Strategy for Testing Series
- Sec. 11.8: Power Series (interval and radius of convergence using Ratio test)

You must also be very confident with ALL the material from Calc-I. You can have a look at the following review material from Calc-I:

- [Derivatives] [Practice problems with solutions]
- [Integrals] [Practice problems with solutions]

You must also be very confident with ALL the material covered in previous Mini-tests. You can have a look at the following "Must know material for previous Mini-Tests":

- `http://carretero.sdsu.edu/teaching/M-151/MTs/MiniTest1_must_know.pdf`
- `http://carretero.sdsu.edu/teaching/M-151/MTs/MiniTest2_must_know.pdf`

In addition to studying ALL Calc-I and Calc-II material above, you must be very confident with the following basic and fundamental topics/formulas/techniques/tricks/hints/etc.:

GENERAL:

- If the problem has a constant (like α, A, β, etc.) just carry it through! It is a constant for the problem and it should be left untouched. Think about the way that you carry π around in expressions without using its actual numerical value [in fact it is *much* easier to write π than writing 3.14159265358979323846...].

DIFFERENTIAL EQUATIONS:

- Solve linear, 1st order, DEs by integrating factor: put equation in $y' + Py = Q$ form, multiply BOTH SIDES of DE by integrating factor $I = \exp(\int P)$, and integrate [do not forget your constant!].
- Knowing the difference between the general solutions to the DE (the one with the arbitrary constant) and the particular solution (the one that passes through a specific initial condition).

SEQUENCES and SERIES:

- Sum of geometric series. This always works: $\displaystyle\sum_{n=\ldots}^{\infty} b\, r^{n\pm\cdots} = \frac{\text{first term}}{1-r}$ (provided $|r| < 1$).
- Recognize a pattern in a series and put it in the form of a geometric series.
- Put a number with infinite repetition of decimals digits into a fraction using series.

... continues overleaf ⟶

- Series tests: (Must check **precise conditions** before applying!)
  - Geometric series. ($|r| < 1$)
  - p-test. ($\sum \frac{1}{n^p}$ conv. for $p > 1$)
  - Integral test. (f cont., $f > 0$, $f \downarrow$)
  - Telescopic sum (use partial fractions).
  - Sandwich/Direct Comparison test. ($a_n > 0$ and $b_n > 0$, i.e. need bottom bread slice!)
  - Limit comparison test. ($a_n > 0$ and $b_n > 0$)
  - Alternating series. ($a_n = (-1)^n b_n$, $b_n \downarrow$, $b_n \to 0$)
  - Ratio test. ($L < 1$ conv., $L > 1$ div., $L = 1$ inconclusive)
- Convergence vs. Absolute Convergence vs. Conditionally Convergent. [Abs. Conv. $\Rightarrow$ Conv.]
- Sec. 11.7 (including ALL of its examples and exercises): very much recommended!
- Activity#10: `https://carretero.sdsu.edu/teaching/M-151/labs/lab10_series_applications.pdf`
  - Construct series from a word problem. (cf. compounding interest problems, drug delivery problems, etc... [see exercises with word problems from Sec. 11.2]).
  - Construct a series from a percentage problem [remember that the $p\%$ of a quantity Q is $\frac{p}{100}Q$ and if you compound you get $\frac{p}{100}Q + Q = (1 + \frac{p}{100})Q$].
- Computing interval of convergence for power series (usually using the ratio test).
- Being able to determine the radius of convergence (if only convergent about $a \Rightarrow R = 0$ and if convergent everywhere $\Rightarrow R = \infty$). Remember that R is *half* of the length of the interval of convergence.
- Remember: $|x - A| < B \;\Rightarrow\; -B < x - A < B$ [and thus $-B + A < x < B + A$].
- For power series, ratio test fails at end points $\Rightarrow$ replace x by the end points and check for convergence of the resulting series. Do NOT study convergence at the end points if you are told not to do so!
- If requested to use test X, then use test X and NOT test Y! If you do not use the indicated test you will earn NO points.