Must Know Material for Mini-test#4 - M151 - Calculus II - Spring 2021

This sheet contains a list of the material that MUST be second nature to you. In addition to studying the following Calc-II material that will be included in Mini-test#4:

- Sec. 11.8: Power Series (interval and radius of convergence using Ratio test)
- Sec. 11.9: Representations of Functions as Power Series
- Sec. 11.10: Taylor and Maclaurin Series
- Sec. 11.11: Applications of Taylor Polynomials (no Taylor inequality nor remainder)
- Sec. 10.1: Curves Defined by Parametric Equations
- Sec. 10.2: Calculus with parametric Equations
- Sec. 10.3: Polar Coordinates

You must also be very confident with ALL the material from Calc-I. You can have a look at the following review material from Calc-I:

- [Derivatives] [Practice problems with solutions]
- [Integrals] [Practice problems with solutions]

You must also be very confident with ALL the material covered in previous Mini-tests. You can have a look at the following "Must know material for previous Mini-Tests":

- `http://carretero.sdsu.edu/teaching/M-151/MTs/MiniTest1_must_know.pdf`
- `http://carretero.sdsu.edu/teaching/M-151/MTs/MiniTest2_must_know.pdf`
- `http://carretero.sdsu.edu/teaching/M-151/MTs/MiniTest3_must_know.pdf`

In addition to studying ALL Calc-I and Calc-II material above, you must be very confident with the following basic and fundamental topics/formulas/techniques/tricks/hints/etc.:

GENERAL:

- If the problem has a constant (like α, A, β, etc.) just carry it through! It is a constant for the problem and it should be left untouched. Think about the way that you carry π around in expressions without using its actual numerical value [in fact it is *much* easier to write π than writing 3.14159265358979323846...].

SERIES:

- Given the series of a function $f(x)$ find the series for $A\,x^a\,f(B\,x^b)$, $\frac{d}{dx}[A\,x^a\,f(B\,x^b)]$, and $\int[A\,x^a\,f(B\,x^b)]dx$. Namely, manipulate (derivatives and integrals) of functions expressed by series.
- Ratio test ($L<1$ conv., $L>1$ div., $L=1$ inconclusive) and computing interval of convergence for power series. Remember: $|x-A|<B \;\Rightarrow\; -B<x-A<B$ [and thus $-B+A<x<B+A$].
- Learn formula and how to use it for Maclaurin series: $f(x)=\displaystyle\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}\,x^n$, [where $f^{(n)}$ is n-th derivative]. (do not forget the factorial in the denominator!)
- Learn formula and how to use it for Taylor series: $f(x)=\displaystyle\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}\,(x-a)^n$, [$f^{(n)}$ is n-th derivative]. (do not forget the factorial in the denominator!)
- Know how to interpret a Taylor polynomial of order n, $T_n(x)$, as an approximation of a function at $x=a$.

PARAMETRIC and POLAR:

- Calculus with parametric equations: $\begin{cases} x &= f(t) \\ y &= g(t)\end{cases}$
  - Slope of tangent line to parametric curve: $m=\dfrac{dy}{dx}=\dfrac{dy/dt}{dx/dt}$.
  - Area: $A=\int_{x_1}^{x_2} y(x)\,dx=\int_{t_1}^{t_2} g(t)\,f'(t)\,dt$ where $x_1=f(t_1)$ and $x_2=f(t_2)$.
  - Arclength: $L=\int_{x_1}^{x_2} ds=\int_{t_1}^{t_2}\sqrt{[f'(t)]^2+[g'(t)]^2}\,dt$ where $x_1=f(t_1)$ and $x_2=f(t_2)$.
- Polar coordinates: $r=f(\theta)$
  - Coord: $x=r\cos(\theta)$ and $y=r\sin(\theta)$. Curve: $r=f(\theta)$. In parametric: $\begin{cases} x(\theta) &= f(\theta)\cos(\theta) \\ y(\theta) &= f(\theta)\sin(\theta)\end{cases}$
  - Table of sin and cos for main angles.
  - Being able to solve $\cos(t)$ or $\sin(t)$ equals to main angles. The same for $\cos(n\,t)$ or $\sin(n\,t)$ for any n.
  - Graphing a polar curve $r=f(\theta)$ by first plotting $y=f(\theta)$ in Cartesian. Remember that negative r means that you need to take the symmetric wrt the origin.
  - Use symmetries for polar curves $r=f(\theta)$. Namely: f even $\Rightarrow$ curve up-down symmetric, f odd $\Rightarrow$ curve left-right symmetric, $f(\theta)=f(\pi-\theta)\Rightarrow$ curve left-right symmetric.
  - Convert a graph from polar $r=f(\theta)$ to Cartesian $y=g(x)$. Need to use $\cos(\theta)=x/r$, $\sin(\theta)=y/r$ and $r^2=x^2+y^2$ and eliminate all r's and θ's in favor of x's and y's.