Sec. 11.9: Power Series

7. (6 pts) Using the fact that function $f(x)$ can be written by the following series: $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{4^n}$, find the series representation (using the Σ notation) for the following functions:

(a) (2 pts) $g(x) = 3\,f(x^2)$

$g(x) = \sum_{n=\square}^{\infty} \square$

(b) (2 pts) $h(x) = \frac{d}{dx}\left[4\,x\,f(x^3)\right]$

$h(x) = \sum_{n=\square}^{\infty} \square$

(c) (2 pts) $w(x) = \int x\,f(x^4)\,dx$ [The constant C is already written for you].

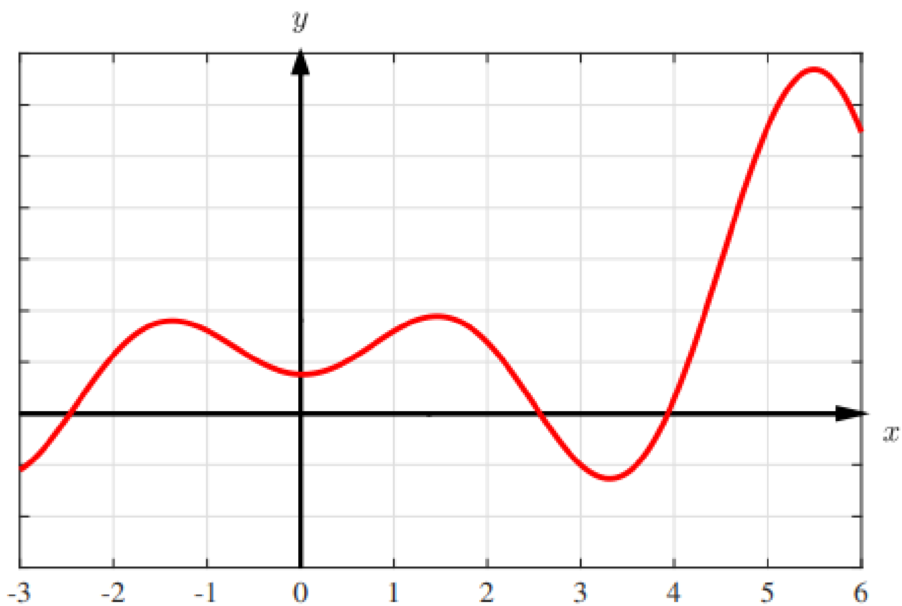
$w(x) = \sum_{n=\square}^{\infty} \square + C$

7. (8 pts) Compute the Taylor polynomial of order 3 (i.e. third degree polynomial) for $\boldsymbol{g(x) = Bx + C\,e^{Ax}}$ about $\boldsymbol{x = 0}$.

$g(x) \approx$

8. (5 pts) Suppose that you are given the graph of the function $f(x)$ depicted on the right. Let us denote $T_n(x)$ the Taylor polynomial approximation of order n [$n = 0$ denotes a constant, $n = 1$ denotes a LINEAR approximation, $n = 2$ denotes a QUADRATIC approximation, etc...]. Sketch the graphs of the following Taylor approximations:

(a) T_0 at $\boldsymbol{x = 0}$ (use a **thin solid** line).
(b) T_2 at $\boldsymbol{x = 0}$ (use a **dashed** line).
(c) T_0 at $\boldsymbol{x = 3}$ (use a **thin solid** line).
(d) T_1 at $\boldsymbol{x = 3}$ (use a **dashed** line).
(e) T_2 at $\boldsymbol{x = 5.5}$ (use a **dashed** line).



Sec. 10.1: Curves Defined by Parametric Equations
Sec. 10.2: Calculus with parametric Equations (tangents, areas, arclength

Find an equation ($y = ...$) of the tangent to the curve at the given point.

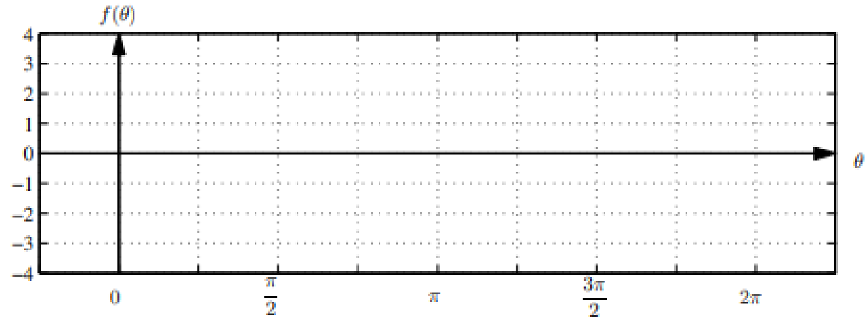
$$x = \cos t + \cos 2t,\ \ y = \sin t + \sin 2t,\ \ (x, y) = (-1, 1)$$

Find the exact length of the curve. $x = 1 + 3t^2, \quad y = 4 + 2t^3, \quad 0 \leqslant t \leqslant 1$

Find the area enclosed by the x-axis and the curve $x = t^3 + 1, y = 2t\text{-}t^2$

Sec. 10.3: Polar Coordinates

Let $\boldsymbol{r = f(\theta) = 4\sin(\theta)}$



(A) Sketch the graph of $r = f(\theta)$ for $0 \leq \theta \leq 2\pi$ in CARTESIAN coordinates and identify ALL minima and maxima.

$x = r\cos\theta$ [redacted]

$y = r\sin\theta$ [redacted]

(C)-(D): Use the fact that the slope in parametric is: $m = \dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ [redacted] ______________________________

(C) Find all (r, θ) points where the curve as a HORIZONTAL tangent.

(D) Find all (r, θ) points where the curve as a VERTICAL tangent.

(A) Sketch the graph of $r = f(\theta)$ for $0 \leq \theta \leq 2\pi$ in CARTESIAN coordinates and identify ALL minima and maxima.