Mixing Problems — Math 151 — Calculus II — Spring 2021

1. Consider a 20 L bucket initially filled to the brim with fresh water. Sea water containing 20 g of salt per liter is pumped at a rate of 5 L/min and is allowed to drain from the brim (i.e., the bucket is always completely full). The liquid is maintained well mixed at all times.

(a) Write a differential equation AND its initial condition for the **AMOUNT OF SALT IN THE BUCKET** $y(t)$ **(in grams)** as a function of time t.

Diff. Eq.: , Initial Condition:

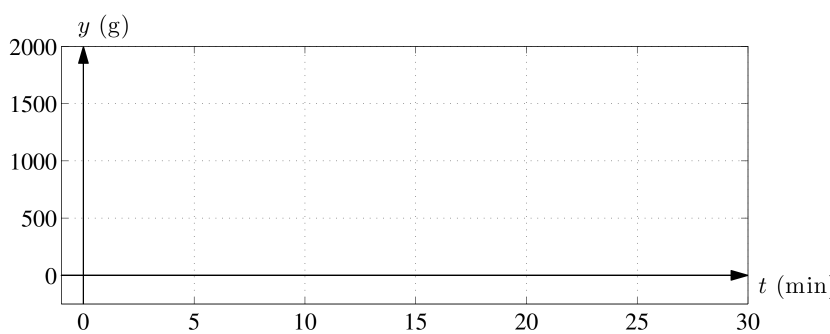
(b) Find the general solution to this differential equation.

General sol: $y(t) =$

(c) Find the particular solution satisfying the initial condition.

Particular sol: $y(t) =$

(d) of the differential equation you found in (a) and **(ii)** include a sketch (in bold) of the solution you obtained in (c) above.



(e) What will be the amount of salt M (in grams) in the bucket after a very long time? Explain!

$M =$

2. Mixing problem: Dialysis treatment removes urea (or other waste products) from a patient's body by diverting some blood flow to a filter that completely filters out the urea such that a clean blood flow (without urea) is channelled back into the patient while the patient's total blood volume is kept constant. An average individual has **5 liters** of blood and a standard dialysis flow is **2 liters per minute**. Suppose that the patient starts the dialysis treatment with a concentration of urea of **3 grams/liter**.

(a) Write a differential equation & its initial condition [i.e. the total amount of urea (in grams) in the patient at the beginning of the dialysis] for the **TOTAL AMOUNT OF UREA** in the patient $y(t)$ (in grams) as a function of time t (in minutes).

(b) Find the general solution to this differential equation.

Diff. Eq.:	, Iinitial Condition:

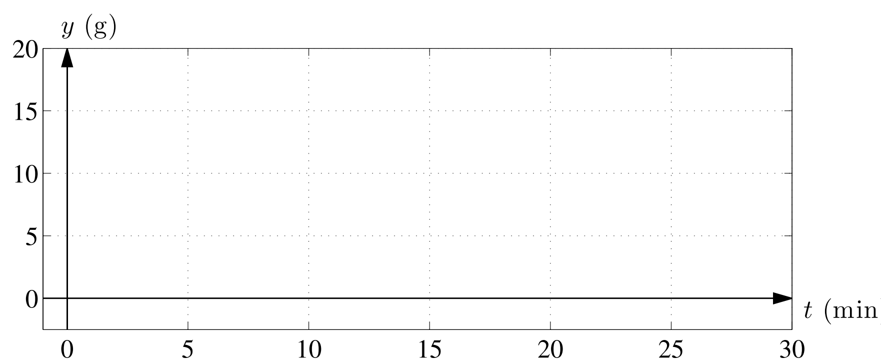
$y(t) =$

(c) Find the particular solution satisfying the initial condition.

$y(t) =$

(d) **(i)** Draw a sketch for the direction field of the differential equation you found in (a) and **(ii)** include a sketch (in bold) of the solution you obtained in (c) above.

(e) **(i)** What is the amount of urea (in grams) left inside the patient after 1/2 hour of dialysis?
(ii) How long would it take for the amount of urea to be half of the amount before treatment?
(iii) According to the solution found in **(c)**, how much urea will be left in the patient if the dialysis continues forever? Explain!



3. A tank with **400 liters** of beer initially contains **5%** alcohol (by volume). Beer with **4%** alcohol is pumped into the tank at a rate of **3 L/min** and the mixture is pumped out at the same rate.

(a) Write a differential equation and its initial condition for the percentage $\boldsymbol{p(t)}$ of alcohol in the tank ($\boldsymbol{t}$ is measured in minutes).

Diff. Eq.: , Ininitial Condition:

(b) Find the general solution to this differential equation.

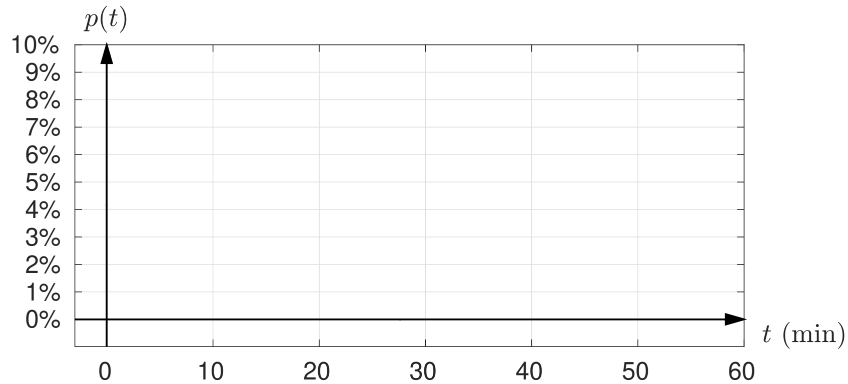
$p(t) =$

(c) Find the particular solution satisfying the initial condition.

$p(t) =$

(d) **(i)** Draw a sketch for all possible solutions to this differential equation and **(ii)** include a sketch (in bold) of the solution obtained in **(c)**.

(e) **(i)** What is the percentage of alcohol in the
tank after 1/2 hour?
(ii) According to the solution found in **(c)**, what is the alcohol percentage if we wait forever? Explain! Why?



4. A tank with **1000 liters** of water initially contains **4 g** of calcium. Water with a concentration of **2 mg/L** (milligrams/liter) is pumped into the tank at a rate of **1 L/min** and the mixture is pumped out at the same rate.

(a) Write a differential equation and its initial condition for, $\boldsymbol{y(t)}$, the TOTAL amount of calcium **in grams** (not milligrams!) in the tank. ($\boldsymbol{t}$ is measured in minutes. Note: **1g = 1,000 mg**).

Diff. Eq.: , Initial Condition:

(b) Find the general solution to this differential equation.

$y(t) =$

(c) Find the particular solution satisfying the initial condition.

$y(t) =$

(d) **(i)** Draw a sketch for all possible solutions to this differential equation and **(ii)** include a sketch (in bold) of the solution obtained in **(c)**.

(e) **(i)** What is the amount of calcium in the tank after 1/2 hour?
(ii) According to the solution found in **(c)**, what is the amount of calcium if we wait forever? Explain! Why?