Midterm #1 (v1) — Math 151 — Calculus II — Fall 2017

Professor/TA: ______________________ **Sec:** ________ **RedID:** ______________

NAME (printed): ______________________ ______________________

(Last Name) **(First Name)**

I, ______________________, pledge that this material is completely my own work, and that I did not take, borrow, or copy any portions from any other person(s). I understand if I violate this honesty pledge, I am subject to disciplinary actions pursuant to the appropriate sections of the San Diego State University Policies.

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Signature

(1) Do NOT open this test booklet until told to do so.
(2) Do ALL your work on this test booklet.
(3) If you need extra space please use the back of the LAST page.
(3') If you need extra space please ask instructor for extra paper.
(4) NO CALCULATORS, NO CHEAT-SHEETS or any other aids allowed.
(5) You may write in either pen or pencil, but answers deemed illegible will be ignored.
(6) Please enter your answers in the BOXES provided
(7) Please check that all **8 pages** (including this cover sheet and the cheat-sheet at the end) are intact.
(8) The value for each question is given in the table below.
(9) In all the questions you should indicate how you arrived at your answer.
(10) To get full credit you need to simplify your answers (cf. $\sin(0) = 0, e^0 = 1, \sqrt{4} = 2, 2/4 = 1/2$, etc...).

1	2	3	4	5	6	7	8	9	Total
/10	/10	/10	/10	/7	/8	/10	/20	/15	/100

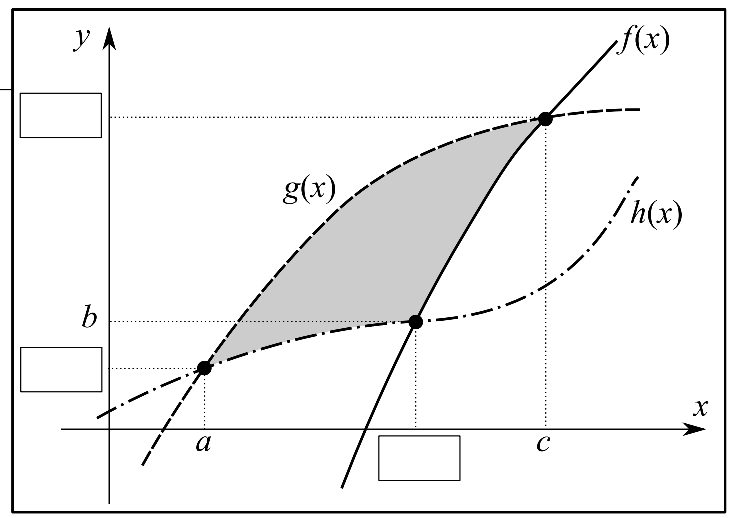
1. (10 pts) Hyperbolic functions

a) (6 pts) (i) **Using the definition** of $\sinh(x)$ and $\cosh(x)$ in term of exponentials, find the derivative of $\boldsymbol{f(x) = 3\sinh(4x)}$.
(ii) Check that you get the right result from **direct** differentiation of hyperbolic functions.

b) (4 pts) Compute the average of $\boldsymbol{f(x) = 2\sinh(5x+3) + \cosh(x)}$ over the interval $\boldsymbol{0 \leq x \leq 3}$. Simplify your result as much as possible.

$f_{\text{ave}} =$

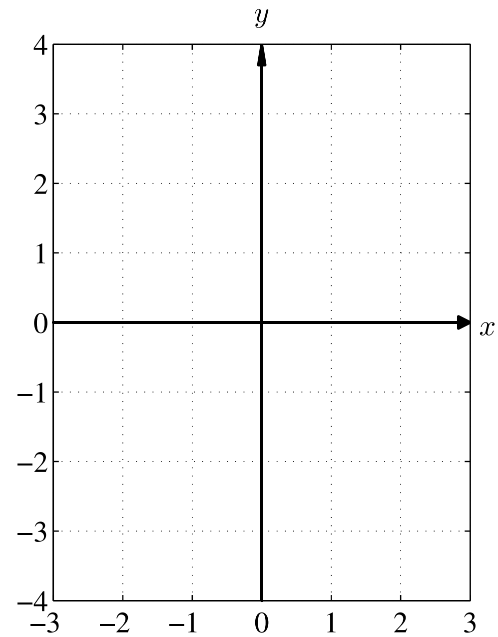
2. (10 pts) Write the integrals for the area defined by the shaded region.
(a) On the plot: fill the empty boxes.
(b) A_x: Write area as integral(s) with respect to x and
(c) A_y: Write area as integral(s) with respect to y.



$A_x =$

$A_y =$

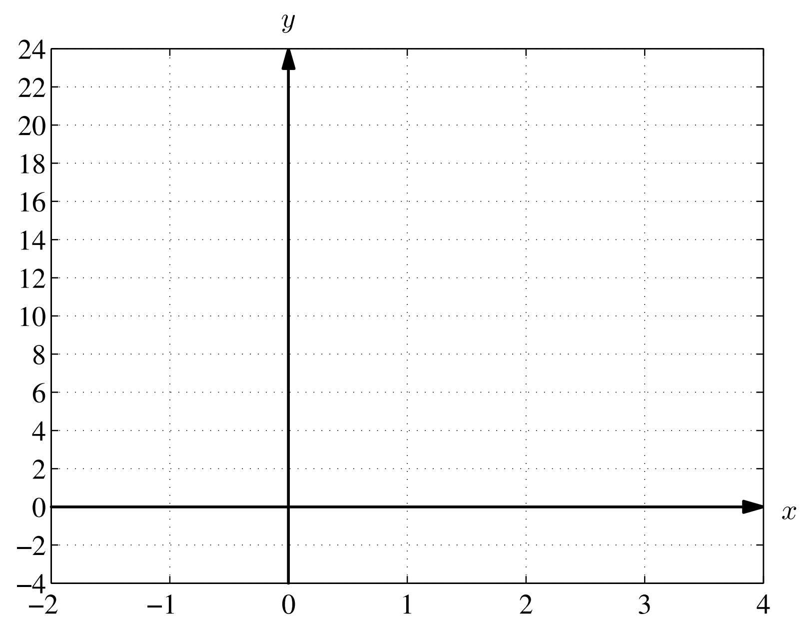
3. (10 pts) Using **WASHERS** write an integral (or integrals) for the solid generated by rotating about the $\boldsymbol{y}$**-axis** the region, in the FIRST quadrant, delimited by the graphs of the following functions: $\boldsymbol{x = 1}$, $\boldsymbol{y = 3}$, and $\boldsymbol{y = x^2 - 1}$.
(a) Sketch the functions and find their intersections, **(b)** sketch (i) the solid, (ii) the region, and (iii) a typical washer.
NOTE: you only need to write the integral but you do not need to compute it!



Intersection pts (order by increasing x and then y): $P_1 = (\quad , \quad)$ $P_2 = (\quad , \quad)$ $P_3 = (\quad , \quad)$

$V_1 = \int_{\square}^{\square} \boxed{\quad} \, d\square$

4. (10 pts) Using volumes by **SHELLS**, write an explicit integral for the solid generated by rotating about the **line** $\boldsymbol{x = 1}$ (note that the line is **off-axis**!) the region delimited by the graphs of: $\boldsymbol{y = e^x + 1}$ and $\boldsymbol{y = 0}$ for $\boldsymbol{1 \le x \le 3}$.
(a) Sketch the functions, **(b)** the region, **(c)** the solid and **(d)** a typical shell.
Note 1: you only need to write the integral but you do not need to compute it!
Note 2: for simplicity use the following approximations: $e^1 \approx 3$, $e^2 \approx 7$, and $e^3 \approx 20$



$V_2 = \int_{\square}^{\square} \boxed{\quad} \, d\square$

5. (7 pts) Compute the following integral

$I_1 = \int 2\,x\,\exp(3x^2 - 2)\,dx$, [Remember that $\exp(x) = e^x$].

$I_1 =$

6. (8 pts) Compute the following integral

$I_2 = \int \cos^3(x)\sin^4(x)\,dx$

$I_2 =$

7. (10 pts) A construction worker weighing **100 Kg** has to start from the bottom of a building that is **20 m** high were a heavy cable weighing **2 Kg per meter** is hanging. As the worker climbs a ladder, he collects the cable until he (and the entire cable) reaches the top. What is the total amount of work done when performing this task?
Note 1: do NOT forget that the worker has to pull his own body weight up as well.
Note 2: the force due to gravity is $F = mg$ and consider, for simplicity, that $g = 10$ m/s^2.
Hint: first compute the mass m as the combined mass of the worker plus the collected cable as a function of height.

8. (20 pts) Compute the following integrals

a) (10 pts) $I_3 = \int 2\,t^2\,e^{3t}\,dt$

$I_3 =$

b) (10 pts) $I_4 = \int_1^2 (x+2)\,\ln(x)\,dx$

$I_4 =$

9. (15 pts) Trigonometric substitution. **NOTE: you cannot leave your result as a composition of an inverse trig. function inside a trig. function (or vice-versa). A single inverse trig. is ok.**

a) (10 pts) Compute: $I_5 = \int \frac{x^3}{\sqrt{4-x^2}}\,dx$

$I_5 =$ ☐

b) (5 pts) Using trig. substitution, write the following integral as a trigonometric integral, i.e. rewrite it as $I_6 = \int_{\theta_1}^{\theta_2} f(\theta)\,d\theta$.
Notes: Clearly state the trigonometric substitution. Do NOT evaluate the integral, just write it as $I_6 = \int_{\theta_1}^{\theta_2} f(\theta)\,d\theta$.
$I_6 = \int_{-1}^{0} \sqrt{x^2+2x+2}\;dx =$

$I_6 = \int_{☐}^{☐}$ ☐ $d\theta$

This cheat sheet contains some formulas that you might find useful.

- Trigonometric identities/formulas:
  - $\sin(x \pm y) = \sin x \, \cos y \pm \cos x \, \sin y$
  - $\cos(x \pm y) = \cos x \, \cos y \mp \sin x \, \sin y$
  - $\sin 2x = 2 \sin x \, \cos x$
  - $\cos 2x = 1 - 2\sin^2 x$
  - $\sin^2 x = \dfrac{1 - \cos 2x}{2}$
  - $\cos^2 x = \dfrac{1 + \cos 2x}{2}$
  - $\sin A \, \cos B = \frac{1}{2}[\sin(A - B) + \sin(A + B)]$
  - $\sin A \, \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$
  - $\cos A \, \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$

- Integrals/derivatives involving trigonometric functions:
  - $[\tan x]' = \sec^2 x$
  - $[\sec x]' = \sec x \, \tan x$
  - $[\arcsin x]' = [\sin^{-1} x]' = \dfrac{1}{\sqrt{1 - x^2}}$
  - $[\arccos x]' = [\cos^{-1} x]' = -\dfrac{1}{\sqrt{1 - x^2}}$
  - $[\arctan x]' = [\tan^{-1} x]' = \dfrac{1}{1 + x^2}$
  - $[\operatorname{arccot} x]' = [\cot^{-1} x]' = -\dfrac{1}{1 + x^2}$
  - $[\operatorname{arcsec} x]' = [\sec^{-1} x]' = \dfrac{1}{x\sqrt{x^2 - 1}}$
  - $[\operatorname{arccsc} x]' = [\csc^{-1} x]' = -\dfrac{1}{x\sqrt{x^2 - 1}}$
  - $\int \tan x \, dx = \ln|\sec x|$
  - $\int \sec x \, dx = \ln|\sec x + \tan x|$

- Integrals/derivatives involving hyperbolic functions:
  - $[\tanh x]' = \operatorname{sech}^2 x$
  - $[\operatorname{sech} x]' = -\operatorname{sech} x \, \tanh x$
  - $[\operatorname{arcsinh} x]' = [\sinh^{-1} x]' = \dfrac{1}{\sqrt{1 + x^2}}$
  - $[\operatorname{arccosh} x]' = [\cosh^{-1} x]' = \dfrac{1}{\sqrt{x^2 - 1}}$
  - $[\operatorname{arctanh} x]' = [\tanh^{-1} x]' = \dfrac{1}{1 - x^2}$
  - $[\operatorname{arcsech} x]' = [\operatorname{sech}^{-1} x]' = -\dfrac{1}{x\sqrt{1 - x^2}}$
  - $\int \tanh x \, dx = \ln(\cosh x)$
  - $\int \operatorname{sech} x \, dx = \tan^{-1}|\sinh x|$

Use this page as extra space. Write your: Name: ______________________ Sec: ______ RedID: ________________