Midterm #1 (v3) — Math 151 — Calculus II — Fall 2018

Professor/TA: ______________________ **Sec:** ________ **RedID:** ______________

NAME (printed): ______________________ ______________________

(Last Name) **(First Name)**

I, ______________________, pledge that this material is completely my own work, and that I did not take, borrow, or copy any portions from any other person(s). I understand if I violate this honesty pledge, I am subject to disciplinary actions pursuant to the appropriate sections of the San Diego State University Policies.

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Signature

(1) Do NOT open this test booklet until told to do so.
(2) Do ALL your work on this test booklet.
(3) If you need extra space please use the back of the LAST page.
(3') If you need extra space please ask instructor for extra paper.
(4) NO CALCULATORS, NO CHEAT-SHEETS or any other aids allowed.
(5) You may write in either pen or pencil, but answers deemed illegible will be ignored.
(6) Please enter your answers in the BOXES provided
(7) Please check that all **6 pages** are intact.
(8) The value for each question is given in the table below.
(9) In all the questions you should indicate how you arrived at your answer.
(10) To get full credit you need to simplify your answers (cf. $\sin(0) = 0, e^0 = 1, \sqrt{4} = 2, 2/4 = 1/2$, etc...).

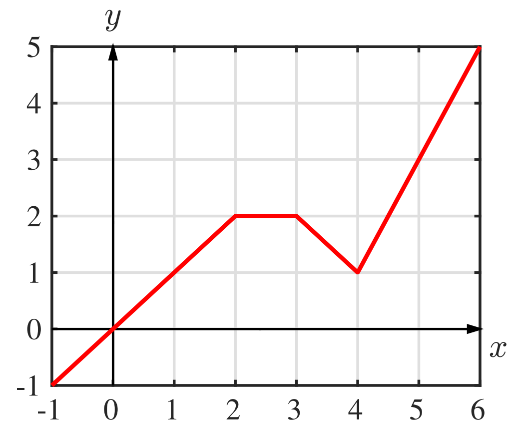
1	2	3	4	5	6	7	8	Total
/10	/10	/10	/10	/10	/10	/20	/14	/94

1. (10 pts) Applications of integrals: average of a function

(a) The linear density of a **15 m** long beam is given by $\boldsymbol{\rho(x) = 10 + \sin(4x)}$ **kg/m**, where x is measured in meters starting from one end. What is the averaged density of the beam?

$\bar{\rho} =$

(b) Find the average of the plotted function on the interval $\boldsymbol{x \in [-1, 6]}$, Express your result as a SINGLE, IRREDUCIBLE, FRACTION!



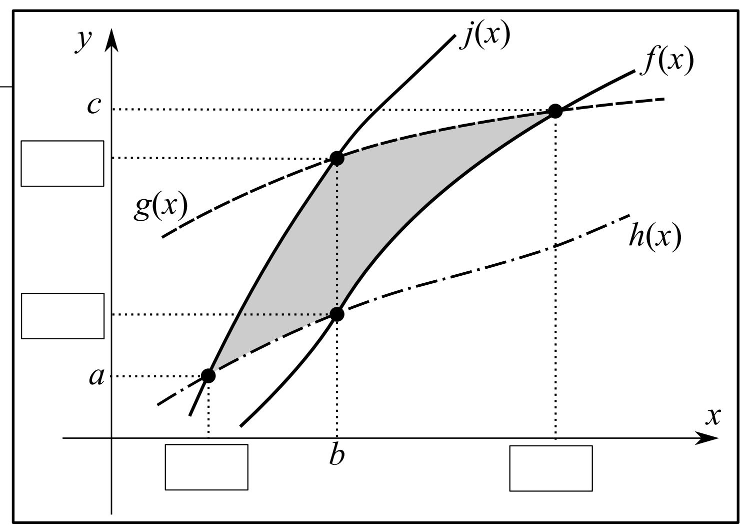
average on $[-1, 6] =$ ———

2. (10 pts) Write the integrals for the area defined by the shaded region.

(a) On the plot: fill the empty boxes.

(b) A_x: Write area as integral(s) with respect to x and

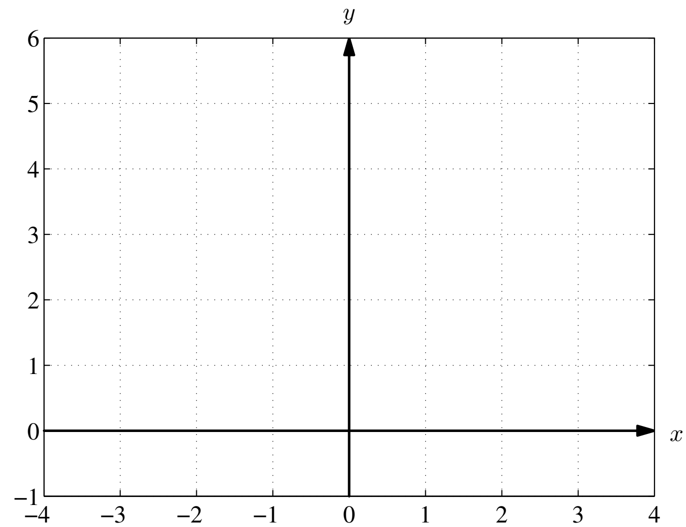
(c) A_y: Write area as integral(s) with respect to y.



$A_x =$

$A_y =$

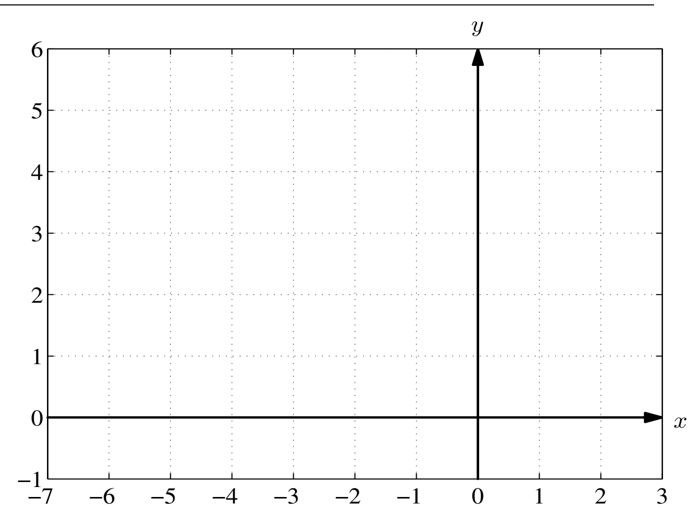
3. (10 pts) Using **WASHERS** write an integral (or integrals) for volume of the solid generated by rotating about the $\boldsymbol{y}$**-axis** the region inside the graphs of the following functions: $\boldsymbol{x = 1}$, $\boldsymbol{y = f(x) = x - 1}$, and $\boldsymbol{y = g(x) = -x + 6}$. **(a)** Sketch the functions and find their intersections, **(b)** sketch (i) the solid, (ii) the region, and (iii) a typical washer for this object. **NOTE: you only need to write the integral but you do not need to compute it!**



Intersection pts (order by increasing x and then y): $P_1 = (\quad , \quad)$ $P_2 = (\quad , \quad)$ $P_3 = (\quad , \quad)$

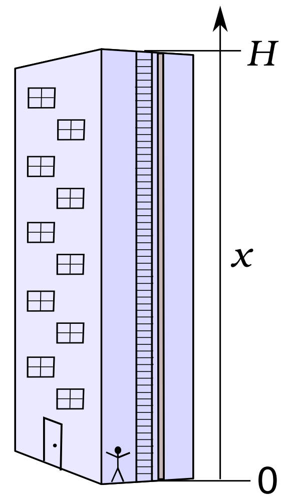
$V_1 = \int_{\square}^{\square} \boxed{\quad} \, d\square + \int_{\square}^{\square} \boxed{\quad} \, d\square$

4. (10 pts) Using volumes by **SHELLS**, write an explicit integral for the solid generated by rotating about the **line** $\boldsymbol{x = -2}$ (note that the line is **off-axis**!) the region delimited by the graphs of: $\boldsymbol{y = x^2 + 1}$ and $\boldsymbol{y = 0}$ for $\boldsymbol{-2 \leq x \leq 2}$. **(a)** Sketch the functions, **(b)** the region, **(c)** the solid and **(d)** a typical shell for this object. **Note: you only need to write the integral but you do not need to compute it!**



$V_2 = \int_{\square}^{\square} \boxed{\quad} \, d\square$

5. (10 pts) A worker with a mass of **80 Kg** has to start from the bottom of a building that has a height H =**40 m** were a **4 Kg PER METER** cable is hanging (see diagram). As the worker climbs a ladder, he collects the cable until he (and the entire cable) reaches the top. **Using calculus**, find the total amount of work done when performing this task.
Note 1: Do NOT forget that the worker has to pull his own body weight up as well.
Note 2: The force due to gravity is $F = mg$ and consider, for simplicity, that $g = 10$ m/s^2.
Note 3: Use the notation depicted in the diagram and **write the corresponding integral**.
Hint: first compute the mass m as the combined mass of the worker plus the collected cable as a function of height x.



6. (10 pts) Show, using the method of volume by slices, that the volume of a sphere of radius R is $V = \frac{4}{3}\pi R^3$. **Draw a diagram including a typical slice for this object**. Clearly indicate the function(s) that you are plotting and the interval of integration.

7. (20 pts) Compute the following integrals

a) (7 pts) $I_1 = \int_2^4 t\,\cos(t)\;dt$

$I_1 =$

b) (7 pts) (c is a constant with $c > 1$) $I_2 = \int x^c \ln(x)\,dx$

$I_2 =$

c) (6 pts) $I_3 = \int \cos^4(x)\sin^3(x)\,dx$

$I_3 =$

8. (14 pts) Trigonometric substitution.

a) (7 pts) Using trigonometric substitution, show that, if $-a \le x \le a$, $\displaystyle\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right)$.

b) (7 pts) Using trig. substitution, write the following integral as a trigonometric integral, i.e. rewrite it as $I_4 = \int_{\theta_1}^{\theta_2} f(\theta)\,d\theta$.
Notes: Clearly state the trigonometric substitution. Do NOT evaluate the integral, just write it as $I_4 = \int_{\theta_1}^{\theta_2} f(\theta)\,d\theta$.

$$I_4 = \int_{-2}^{2} \sqrt{x^2 + 4x + 20}\; dx =$$

$$I_4 = \int_{\square}^{\square} \boxed{}\; d\theta$$

Extra credit (5 pts) Similar to **8.a** above, but using the HYPERBOLIC substitution $x = a\,\sinh(t)$, show that

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \sinh^{-1}\left(\frac{x}{a}\right).$$