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Midterm #1 (v1) — Math 151 — Calculus II — Fall 2019

I, _____, student of section _____, pledge that this material is completely my own work, and that I did not take, borrow, or copy any portions from any other person(s). I understand if I violate this honesty pledge, I am subject to disciplinary actions pursuant to the appropriate sections of the San Diego State University Policies.

_____

Signature

(0) Write your first and last name above using **LARGE CAPITAL LETTERS**: 

(1) If you use pencil please **use pressure!!!**
If you write softly with pencil the scan will be unreadable and your test will NOT be graded!
(2) Do NOT alter the QR-code above! If you do so, your paper will not be graded and you will get a ZERO.
(3) Do NOT open this test booklet until told to do so.
(4) Do ALL your work on this test booklet.
(5) If you need extra space please use the last page.
(6) NO CALCULATORS, NO CHEAT-SHEETS or any other aids allowed.
(7) You may write in either pen or pencil, but answers deemed illegible will be ignored. (see point#1 above)
(8) Please enter your answers in the BOXES provided
(9) Please check that all **8 pages** (including this cover sheet and the extra space page at the end) are intact.
(10) The value for each question is given in the table below.
(11) In all the questions you should indicate how you arrived at your answer.
(12) To get full credit you need to simplify your answers (cf. $\sin(0) = 0, e^0 = 1, \sqrt{4} = 2, 2/4 = 1/2$, etc...).

1	2	3	4	5	6	7	8	Total
/ 6	/ 4	/10	/10	/ 8	/10	/10	/20	/78

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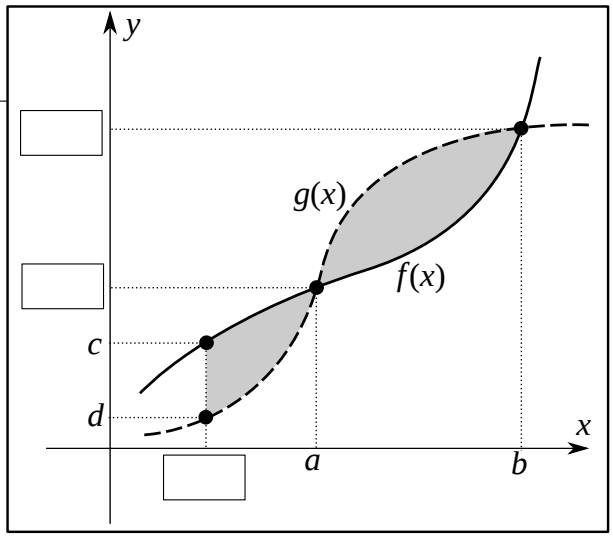
1. (6 pts) Write the integrals for the area defined by the shaded region.
(a) On the plot: fill the empty boxes.
(b) A_x: Write area as integral(s) with respect to x and
(c) A_y: Write area as integral(s) with respect to y.



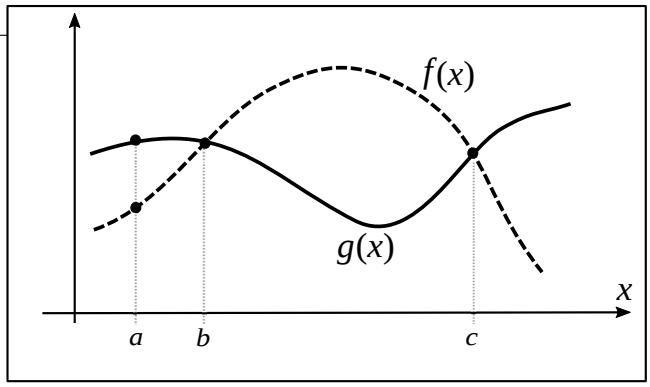
$A_x =$

$A_y =$

2. (4 pts) Applications of integrals: averages.

For the functions depicted to the right, write the corresponding integrals for the **average vertical distance** between the graphs of the functions. Perform this for the following intervals:

(a) Average vertical distance on $a \leq x \leq b$
(b) Average vertical distance on $b \leq x \leq c$
(c) Average vertical distance on $a \leq x \leq c$

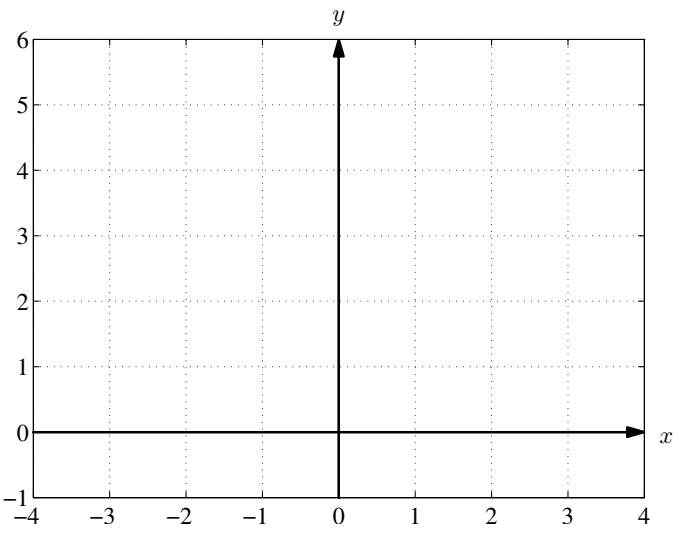


(a) ave. on $[a, b]$:

(b) ave. on $[b, c]$:

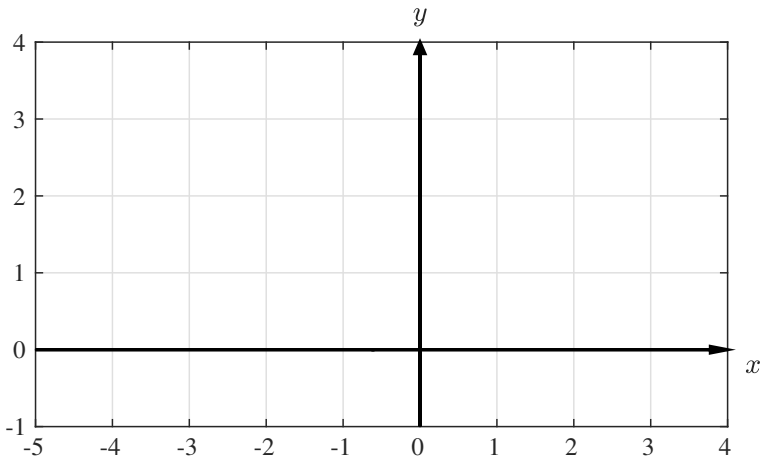
(b) ave. on $[a, c]$:

3. (10 pts) Using **WASHERS** write an integral (or integrals) for volume of the solid generated by rotating about the **y-axis** the region inside the graphs of the following functions: $\boldsymbol{x = 1}$, $\boldsymbol{y = f(x) = x}$, and $\boldsymbol{y = g(x) = -x + 6}$. Sketch (i) the solid, (ii) the region, and (iii) a typical washer for this object.
NOTE: you only need to write the integral(s) but you do not need to compute it!



$V_1 =$

4. (10 pts) Using volumes by **SHELLS**, write an explicit integral for the solid generated by rotating about the **line $x = -1$** (note that the line is **off-axis**!) the region delimited by the graphs of: $\boldsymbol{y = 2 + \cos\left(\frac{\pi}{2}x\right)}$ and $\boldsymbol{y = 0}$ for $\boldsymbol{-1 \le x \le 2}$.
(a) Sketch the functions, **(b)** the region, **(c)** the solid and **(d)** a typical shell for this object.
Note: you only need to write the integral but you do not need to compute it!



$V_2 = \int_{\square}^{\square}$ $\square$ $d\square$

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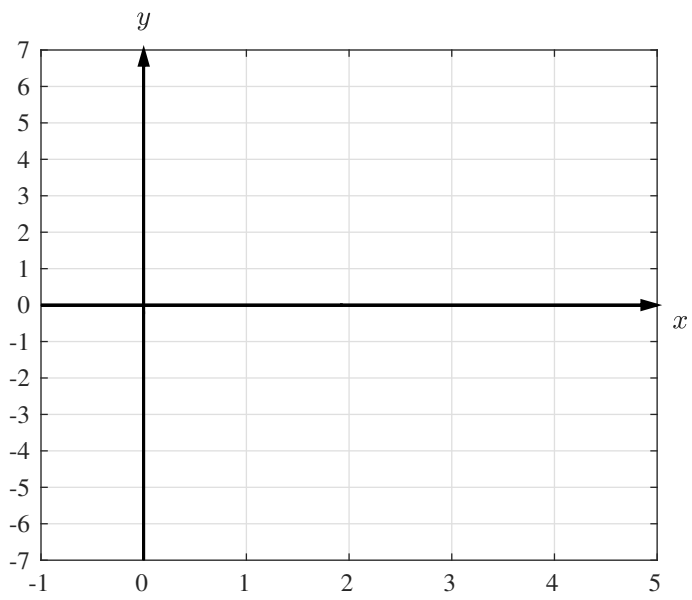
5. (8 pts) (a) Filling the boxes below, describe the volume of revolution that leads, using the method of **WASHERS**, to the integral:

$$\boldsymbol{I = \int_0^4 \pi \left[(x+2)^2 - 4\right]\, dx.}$$

(b) Sketch the functions, the solid and
(c) a typical washer for this object.

The integral I above decribes the volume generated by rotating about the $\boxed{y =}$ axis, the region delimited by the functions $\boxed{y =}$ and $\boxed{y =}$ for $\boxed{\quad \leq x \leq \quad}$.



6. (10 pts) Show, using the method of VOLUME BY **SHELLS**, that the volume of a cone with circular base of radius R and height H is given by $\boldsymbol{V = \frac{1}{3}\pi R^2 H}$. **Draw a diagram including a typical shell for this object**. Clearly indicate the function(s) that you are plotting and the interval of integration.

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7. (10 pts) Work.

a) (3 pts) Compute the work done by the force $F(x) = k\,x$ (i.e., a linear, Hooke's, spring) when moving an object from $x = x_0$ to $x = x_1$.

b) (3 pts) If one can only afford to use **12 Joules** (Joule is the metric unit for work (energy): 1 J = 1 kg $\mathrm{m}^2/\mathrm{s}^2$), how far is it possible to drag an oject to the right, starting at $x = 0$ whose drag force is given by $\boldsymbol{F(x) = 4 + 2x}$? Simplify as much as possible and leave your result as a single number with the correct units.

c) (4 pts) A sack with **30 Kg** of sand is being lifted from $\boldsymbol{y = 0}$ m to $\boldsymbol{y = 20}$ m. The sack has a small hole that allows 100 g of sand to be lost every meter that the sack is pulled up. Compute the total work (energy) necessary to lift the sack.
Hint#1: $F = m \times g$, use $g \approx 10$ m/s^2, 1 Kg = 1,000 g.
Hint#2: First compute the mass as a function of height.

8. (20 pts) Compute the following integrals

a) (4 pts) $\boldsymbol{I_1 = \int \cos^5(x)\sin^2(x)\,dx}$

$I_1 =$

b) (5 pts) $\boldsymbol{I_2 = \int 3x^2\sin(x)\,dx}$

$I_2 =$

c) (3 pts) $\boldsymbol{I_3 = \int 3x\,\cosh(2x^2+1)\,dx}$

$I_3 =$

d) (5 pts) $\boldsymbol{I_4 = \int_0^4 x\,e^{x+2}\,dx}$

$I_4 =$

e) (3 pts) $\boldsymbol{I_5 = \int \cos^4(t)\,dt}$

$I_5 =$

This cheat sheet contains some formulas that you might find useful.

- $\sin(x \pm y) = \sin x \, \cos y \pm \cos x \, \sin y$
- $\cos(x \pm y) = \cos x \, \cos y \mp \sin x \, \sin y$
- $\sin 2x = 2 \sin x \, \cos x$
- $\cos 2x = 1 - 2 \sin^2 x$
- $\sin^2 x = \dfrac{1 - \cos 2x}{2}$
- $\cos^2 x = \dfrac{1 + \cos 2x}{2}$
- $\sin A \, \cos B = \frac{1}{2}[\sin(A - B) + \sin(A + B)]$
- $\sin A \, \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$
- $\cos A \, \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$

Use this space for scratch work...