Midterm #1 (v1) — Math 151 — Calculus II — Spring 2018

Professor/TA: ______________________ **Sec:** ________ **RedID:** ______________

NAME (printed): ______________ ______________
(Last Name) **(First Name)**

I, ________________________, pledge that this material is completely my own work, and that I did not take, borrow, or copy any portions from any other person(s). I understand if I violate this honesty pledge, I am subject to disciplinary actions pursuant to the appropriate sections of the San Diego State University Policies.

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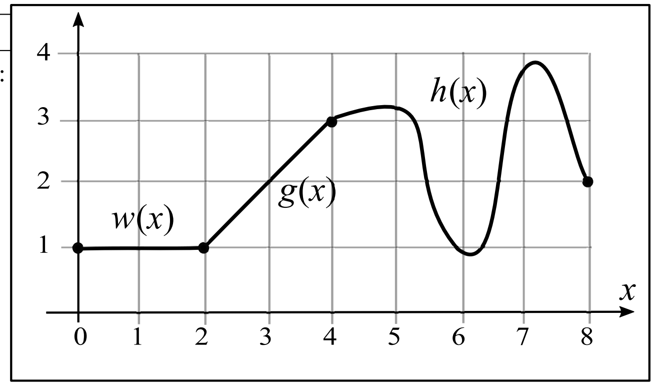
Signature

(1) Do NOT open this test booklet until told to do so.
(2) Do ALL your work on this test booklet.
(3) If you need extra space please use the back of the LAST page.
(3') If you need extra space please ask instructor for extra paper.
(4) NO CALCULATORS, NO CHEAT-SHEETS or any other aids allowed.
(5) You may write in either pen or pencil, but answers deemed illegible will be ignored.
(6) Please enter your answers in the BOXES provided
(7) Please check that all **8 pages** (including this cover sheet and the cheat-sheet at the end) are intact.
(8) The value for each question is given in the table below.
(9) In all the questions you should indicate how you arrived at your answer.
(10) To get full credit you need to simplify your answers (cf. $\sin(0) = 0, e^0 = 1, \sqrt{4} = 2, 2/4 = 1/2$, etc...).

1	2	3	4	5	6	7	8	9	xtr	Total
/5	/5	/10	/10	/10	/10	/10	/20	/10	/5	/90

1. (5 pts) Applications of integrals: average of a function



The function depicted to the right is defined by the following three pieces:

(1) $f(x) = 1$ if $x \in [0, 2]$

(2) $g(x) = x - 1$ if $x \in [2, 4]$

(3) $h(x)$ if $x \in [4, 8]$

(a) What is the average for this function for $x \in [0, 4]$.

(b) What is the average for this function for $x \in [0, 8]$.

Simplify your results as much as possible!

Your final answer has to be exact! (no approximations allowed).

(a) ave. on $[0, 4]$:	**(b)** ave. on $[0, 8]$:

2. (5 pts) Compute the following integral (simplify and leave your result in terms of hyperbolic functions)

$$I_1 = \int_0^1 2\,x\,\cosh(3x^2 - 2)\,dx$$

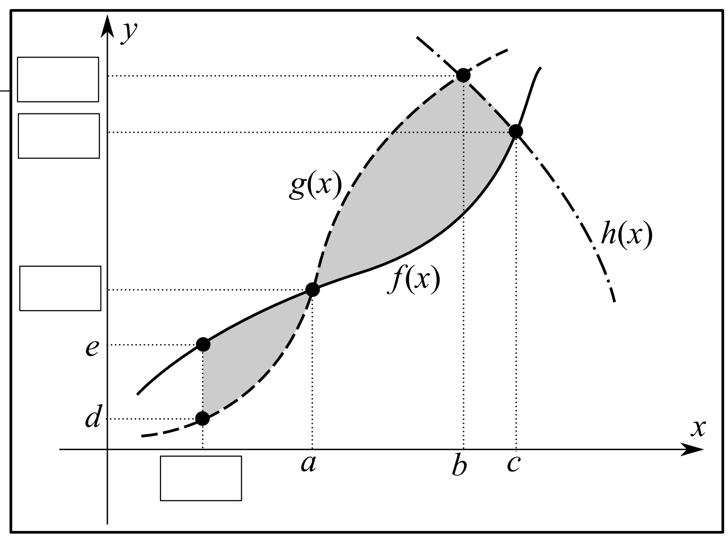
$I_1 =$

3. (10 pts) Write the integrals for the area defined by the shaded region.

(a) On the plot: fill the empty boxes.

(b) A_x: Write area as integral(s) with respect to x and

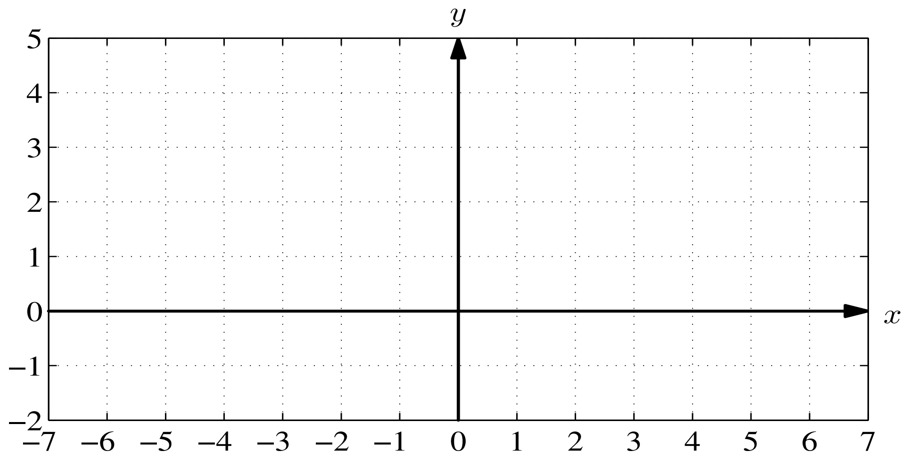
(c) A_y: Write area as integral(s) with respect to y.



$A_x =$

$A_y =$

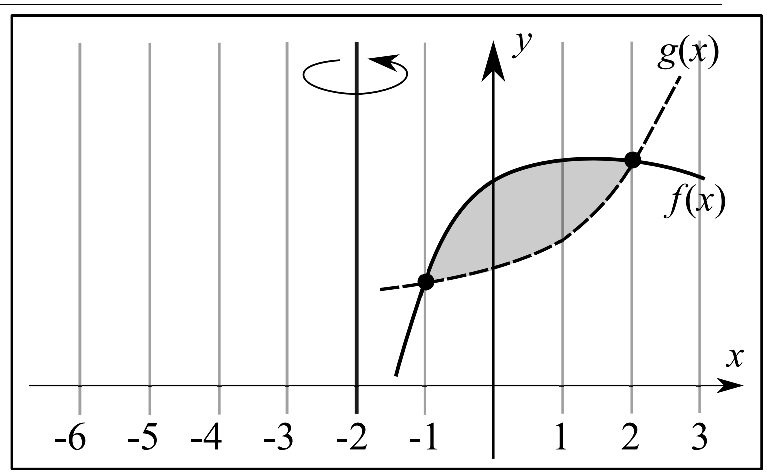
4. (10 pts) Using **WASHERS** write an integral (or integrals) for the solid generated by rotating about the ***y*-axis** the region delimited by the graphs of the following functions: $x = 3$, $y = \frac{1}{3}x + 2$, and $y = x - 2$.
(a) Sketch the functions and find their intersections, **(b)** sketch (i) the solid, (ii) the region, and (iii) a typical washer.
NOTE: you only need to write the integral but you do not need to compute it!



Intersection pts (order by increasing x and then y): $P_1 = (\quad , \quad)$ $P_2 = (\quad , \quad)$ $P_3 = (\quad , \quad)$

$V_1 = \int_{\square}^{\square} \square \, d\square + \int_{\square}^{\square} \square \, d\square$

5. (10 pts) Using the method of volumes by **SHELLS**:
(a) write an integral for the solid generated by rotating about the $x = -2$ **line** (note that the line is off-axis!) the shaded region on the figure (delimited between $y = f(x)$ and $y = g(x)$).
(b) Sketch the solid and **(c)** a typical shell.



$V_2 = \int_{\square}^{\square} \square \, d\square$

6. (10 pts) Work.

a) (5 pts) Compute the work done by the force $F(x) = x + 2\sinh(x) + \cosh(x)$ when moving an object from $x = 0$ to $x = 2$. Simplify as much as possible and leave your result in terms of hyperbolic functions.

b) (5 pts) A rocket with a mass of 3 tons is filled with 40 tons of liquid fuel. In the initial part of the flight fuel burns at a rate of 1 ton per 100 meters of vertical height. How much work (in international units) is done by rocket in the first kilometer of vertical flight? [Hint: use $g \approx 10$ m/s^2, 1 ton = 1,000Kg, 1Km = 1,000m].

7. (10 pts) Show, using the method of volume by slices, that the volume of a cone with circular base of radius R and height h is given by $\boldsymbol{V = \frac{1}{3}\pi R^2 h}$. **Draw a diagram including a typical slice**. Clearly indicate the function(s) that you are plotting and the interval of integration.

8. (20 pts) Compute the following integrals

a) (10 pts) $I_2 = \int 2\,x^2 \cos(x)\,dx$

$I_2 =$

b) (10 pts) $I_3 = \int_0^4 x\,e^{x+2}\,dx$ [Simplify your result as much as possible.]

$I_3 =$

9. (10 pts) Compute the following integrals

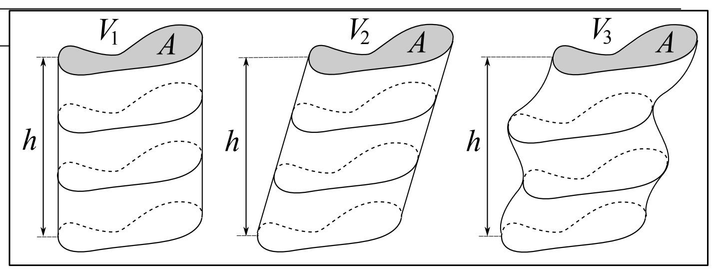
a) (7 pts) $I_4 = \int \cos^5(x)\sin^2(x)\,dx$

$I_4 =$

b) (3 pts) $I_5 = \int A\,\cos(\alpha x)\,\sin(\beta x)\,dx$

$I_5 =$

Extra credit (5 pts) Applications of integrals: volumes



The three volumes on the figure are generated by moving the same cross-sectional area A along different paths for a total height h.

(a) Prove using calculus that the three volumes are the same. Please explain! [no explanation ⇒ no points].

(b) Write **and evaluate** the integral for this volume.

This cheat sheet contains some formulas that you might find useful.

- Trigonometric identities/formulas:
  - $\sin(x \pm y) = \sin x \, \cos y \pm \cos x \, \sin y$
  - $\cos(x \pm y) = \cos x \, \cos y \mp \sin x \, \sin y$
  - $\sin 2x = 2 \sin x \, \cos x$
  - $\cos 2x = 1 - 2\sin^2 x$
  - $\sin^2 x = \dfrac{1 - \cos 2x}{2}$
  - $\cos^2 x = \dfrac{1 + \cos 2x}{2}$
  - $\sin A \, \cos B = \frac{1}{2}[\sin(A - B) + \sin(A + B)]$
  - $\sin A \, \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$
  - $\cos A \, \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$
- Integrals/derivatives involving trigonometric functions:
  - $[\tan x]' = \sec^2 x$
  - $[\sec x]' = \sec x \, \tan x$
  - $[\arcsin x]' = [\sin^{-1} x]' = \dfrac{1}{\sqrt{1 - x^2}}$
  - $[\arccos x]' = [\cos^{-1} x]' = -\dfrac{1}{\sqrt{1 - x^2}}$
  - $[\arctan x]' = [\tan^{-1} x]' = \dfrac{1}{1 + x^2}$
  - $[\operatorname{arccot} x]' = [\cot^{-1} x]' = -\dfrac{1}{1 + x^2}$
  - $[\operatorname{arcsec} x]' = [\sec^{-1} x]' = \dfrac{1}{x\sqrt{x^2 - 1}}$
  - $[\operatorname{arccsc} x]' = [\csc^{-1} x]' = -\dfrac{1}{x\sqrt{x^2 - 1}}$
  - $\int \tan x \, dx = \ln |\sec x|$
  - $\int \sec x \, dx = \ln |\sec x + \tan x|$
- Integrals/derivatives involving hyperbolic functions:
  - $[\tanh x]' = \operatorname{sech}^2 x$
  - $[\operatorname{sech} x]' = -\operatorname{sech} x \, \tanh x$
  - $[\operatorname{arcsinh} x]' = [\sinh^{-1} x]' = \dfrac{1}{\sqrt{1 + x^2}}$
  - $[\operatorname{arccosh} x]' = [\cosh^{-1} x]' = \dfrac{1}{\sqrt{x^2 - 1}}$
  - $[\operatorname{arctanh} x]' = [\tanh^{-1} x]' = \dfrac{1}{1 - x^2}$
  - $[\operatorname{arcsech} x]' = [\operatorname{sech}^{-1} x]' = -\dfrac{1}{x\sqrt{1 - x^2}}$
  - $\int \tanh x \, dx = \ln(\cosh x)$
  - $\int \operatorname{sech} x \, dx = \tan^{-1} |\sinh x|$