Do NOT write ANYTHING above this line!

Midterm #1 (v3) — Math 151 — Calculus II — Spring 2019

I, _____, student of section _____, pledge that this material is completely my own work, and that I did not take, borrow, or copy any portions from any other person(s). I understand if I violate this honesty pledge, I am subject to disciplinary actions pursuant to the appropriate sections of the San Diego State University Policies.

_____

Signature

(0) Write your first and last name above using **LARGE CAPITAL LETTERS**.
(1) If you use pencil please **use pressure!!!**
If you write softly with pencil the scan will be unreadable and your test will NOT be graded!
(2) Do NOT alter the QR-code above! If you do so, your paper will not be graded and you will get a ZERO.
(3) Do NOT open this test booklet until told to do so.
(4) Do ALL your work on this test booklet.
(5) If you need extra space please use the last page.
(6) NO CALCULATORS, NO CHEAT-SHEETS or any other aids allowed.
(7) You may write in either pen or pencil, but answers deemed illegible will be ignored. (see point#1 above)
(8) Please enter your answers in the BOXES provided
(9) Please check that all **8 pages** (including this cover sheet and the extra space page at the end) are intact.
(10) The value for each question is given in the table below.
(11) In all the questions you should indicate how you arrived at your answer.
(12) To get full credit you need to simplify your answers (cf. $\sin(0) = 0, e^0 = 1, \sqrt{4} = 2, 2/4 = 1/2$, etc...).

1	2	3	4	5	6	7	8	9	Total
/5	/10	/10	/10	/10	/10	/10	/20	/15	/100

1. (5 pts) Applications of integrals: average of a function

The function depicted to the right is defined by the following three pieces:

(1) $w(x) = 1$ if $x \in [0, 2]$

(2) $g(x) = x - 1$ if $x \in [2, 4]$

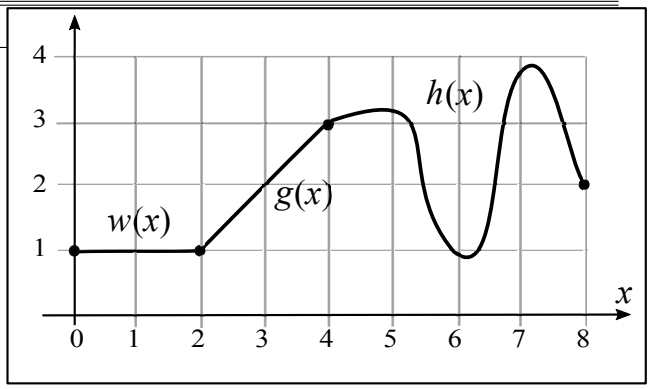
(3) $h(x)$ if $x \in [4, 8]$

(a) Write an integral for the average of this function for $x \in [0, 4]$.

(b) Write an integral for the average of this function for $x \in [0, 8]$.

Simplify your results as much as possible!

Your final answer has to be exact! (no approximations allowed).



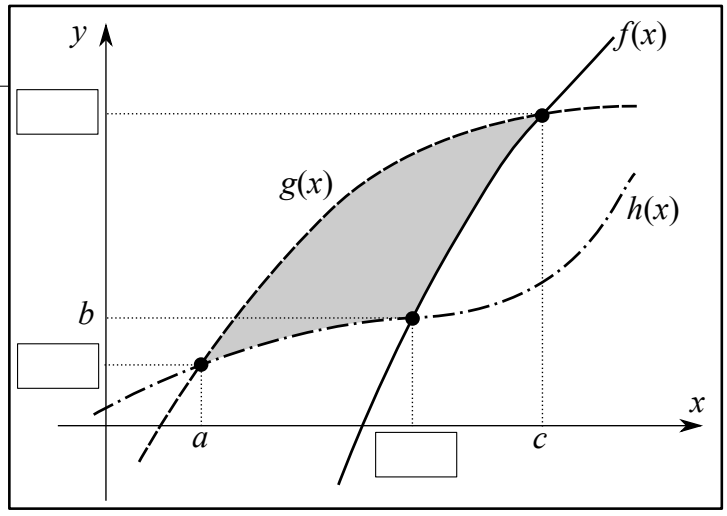
(a) ave. on $[0, 4]$:	**(b)** ave. on $[0, 8]$:

2. (10 pts) Write the integrals for the area defined by the shaded region.

(a) On the plot: fill the empty boxes.

(b) A_x: Write area as integral(s) with respect to x and

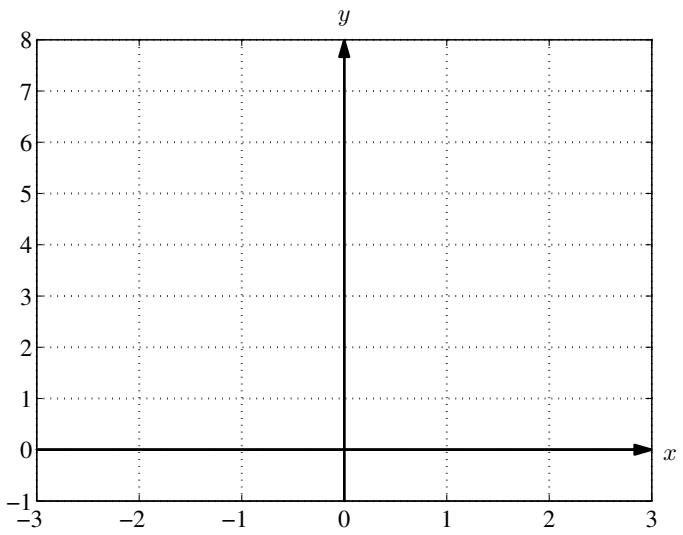
(c) A_y: Write area as integral(s) with respect to y.



$A_x =$

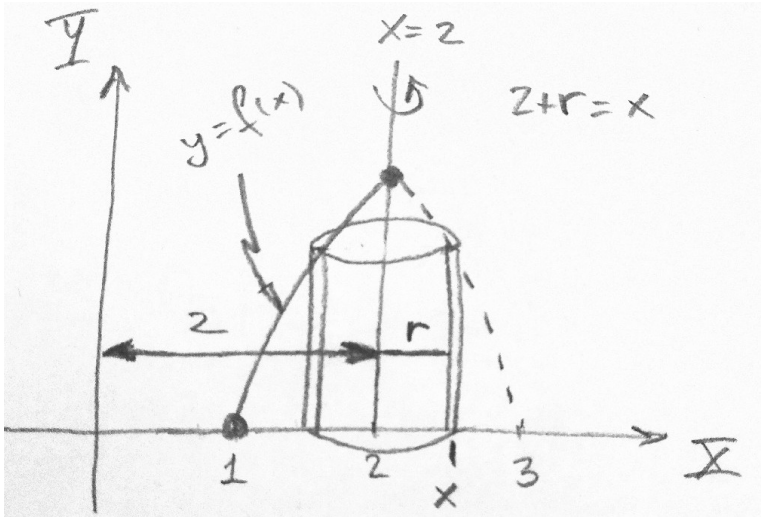
$A_y =$

3. (10 pts) Using **WASHERS** write an integral (or integrals) for volume of the solid generated by rotating about the ***y*-axis** the region inside the graphs of the following functions: $y = 0$, $y = x^2 + 3$, and $0 \leq x \leq 2$.
Sketch (i) the solid, (ii) the region, and (iii) a typical washer for this object.
NOTE: you only need to write the integral(s) but you do not need to compute it!



$V_1 =$

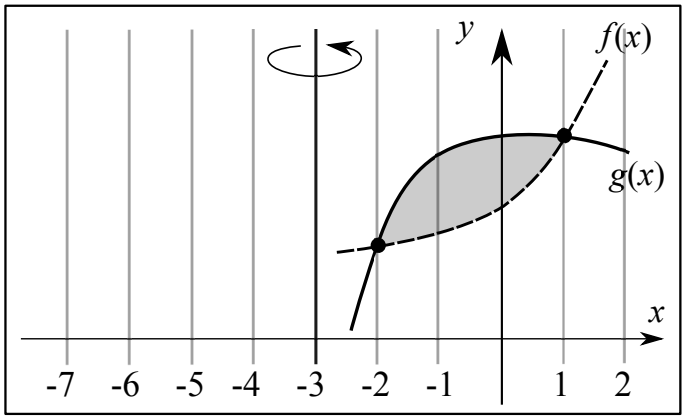
4. (10 pts) A student was asked to use SHELLS to write an integral for the volume generated, by rotating around the $x = 2$ **axis**, the region under the function $y = f(x)$ for $1 \leq x \leq 2$. He produced the drawing below.
(a) What is **WRONG** with the student's drawing that eventually lead him to the incorrect answer?
Just saying "'*this*' should be '*that*' instead of '*this*'" is NOT enough ! Explain in detail, elaborate !!!
(b) Draw the correct sketch, including a typical shell, and write the correct integral for this problem.



$V_2 = \int_{\square}^{\square}$ $\qquad$ $d\square$

Do NOT write ANYTHING above this line!

5. (10 pts) Using the method of volumes by **SHELLS**:
(a) write an integral for the solid generated by rotating about the **$x = -3$ line** (note that the line is off-axis!) the shaded region on the figure [delimited between $y = f(x)$ and $y = g(x)$].
(b) Sketch the solid and **(c)** a typical shell for this object.



$V_3 = \int_{\square}^{\square} \qquad d\square$

6. (10 pts) Show, using the method of volume by slices, that the volume of a pyramid of height H with a RECTANGULAR base of size A by B is given by $\boldsymbol{V = \frac{1}{3} A \times B \times H}$. **Draw a diagram including a typical slice for this object**. Clearly indicate the function(s) that you are plotting and the interval of integration.

7. (10 pts) Work.

a) (5 pts) Compute the work done by the force $\boldsymbol{F(x) = x + 4\,\sinh(x) + \cosh(x)}$ when moving an object from $\boldsymbol{x = 0}$ to $\boldsymbol{x = 4}$. Simplify as much as possible and leave your result in terms of hyperbolic functions.

b) (5 pts) A rocket with a mass of **4 tons** is filled with **20 tons** of liquid fuel. In the initial part of the flight fuel burns at a rate of **1 ton per 100 meters** of vertical height. How much work (in international units) is done by rocket in the **first kilometer** of vertical flight? [Hints: $F = m \times g$, use $g \approx 10$ m/s^2, 1 ton = 1,000Kg, 1Km = 1,000m].

8. (20 pts) Compute the following integrals

a) (5 pts) $\boldsymbol{I_1 = \int \cos^4(x)\,\sin^5(x)\,dx}$

$I_1 =$

b) (6 pts) $\boldsymbol{I_2 = \int 4\,x^2\,e^x\,dx}$

$I_2 =$

c) (6 pts) $\boldsymbol{I_3 = \int_1^2 3\,x\,\ln(x)\,dx}$.

$I_3 =$

d) (3 pts) $\boldsymbol{I_4 = \int \cos^2(t)\,dt}$

$I_4 =$

9. (15 pts) Trigonometric substitution.

a_1) (8 pts) Evaluate the following integral using **TRIGONOMETRIC SUBSTITUTION**.
Yes, I know, this integral can also be done by u-sub (see a_2 next). However, do it using TRIG SUB!

$$\boldsymbol{I_5 = \int \frac{x}{\sqrt{16 - x^2}}\, dx}$$

$I_5 =$

a_2) (2 pts) Now, compute the integral I_5 using u-sub and compare your result with the previous method.

$I_5 =$

b) (5 pts) Using trig. substitution, write the following integral as a trigonometric integral, i.e. rewrite it as $I_6 = \int_{\theta_1}^{\theta_2} f(\theta)\, d\theta$.
Notes: Clearly state the trigonometric substitution. Do NOT evaluate the integral, just write it as $I_6 = \int_{\theta_1}^{\theta_2} f(\theta)\, d\theta$.

$$\boldsymbol{I_6 = \int_2^3 \sqrt{x^2 - 4x + 5}\, dx =}$$

$I_6 = \int$ $\quad d\theta$

This cheat sheet contains some formulas that you might find useful.

- $\sin(x \pm y) = \sin x \, \cos y \pm \cos x \, \sin y$
- $\cos(x \pm y) = \cos x \, \cos y \mp \sin x \, \sin y$
- $\sin 2x = 2 \sin x \, \cos x$
- $\cos 2x = 1 - 2 \sin^2 x$
- $\sin^2 x = \dfrac{1 - \cos 2x}{2}$
- $\cos^2 x = \dfrac{1 + \cos 2x}{2}$
- $\sin A \, \cos B = \frac{1}{2}[\sin(A - B) + \sin(A + B)]$
- $\sin A \, \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$
- $\cos A \, \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$

Use this space for scratch work...