Midterm #2 (v1) — Math 151 — Calculus II — Fall 2017

Professor/TA: ______________________ **Sec:** ________ **RedID:** ______________

NAME (printed): ______________________ ______________________

(Last Name) **(First Name)**

I, ______________________, pledge that this material is completely my own work, and that I did not take, borrow, or copy any portions from any other person(s). I understand if I violate this honesty pledge, I am subject to disciplinary actions pursuant to the appropriate sections of the San Diego State University Policies.

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Signature

(1) Do NOT open this test booklet until told to do so.
(2) Do ALL your work on this test booklet.
(3) If you need extra space please use the back of the LAST page.
(3') If you need extra space please ask instructor for extra paper.
(4) NO CALCULATORS, NO CHEAT-SHEETS or any other aids allowed.
(5) You may write in either pen or pencil, but answers deemed illegible will be ignored.
(6) Please enter your answers in the BOXES provided
(7) Please check that all **6 pages** (including this cover sheet and the extra space page at the end) are intact.
(8) The value for each question is given in the table below.
(9) In all the questions you should indicate how you arrived at your answer.
(10) To get full credit you need to simplify your answers (cf. $\sin(0) = 0, e^0 = 1, \sqrt{4} = 2, 2/4 = 1/2$, etc...).

1	2	3	4	5	6	7	8	Total
/15	/20	/15	/10	/ 5	/10	/10	/ 5	/90

1. (15 pts) Integrals.

a) (5 pts) Using long division, prove that $\dfrac{x^3-4x+3}{x^2-x-2} = x+1+\dfrac{5-x}{x^2-x-2}$

b) (5 pts) Using **a)** above, integrate: $I_1 = \displaystyle\int \frac{x^3-4x+3}{x^2-x-2}\,dx =$

$I_1 =$

c) (5 pts) Write the partial fraction decomposition for the following integral. Do NOT compute the coefficients of the numerators but you MUST JUSTIFY each term in your decomposition (i.e., repeated/non-repeated, linear, quadratic, ...).

$$I_2 = \int \frac{2x^4-4x^2-6x+4}{(x^2+2)\ (x^2+1)^2\ (x+2)^2\ (x-1)\ (5x-2)}\,dx$$

$I_2 = \displaystyle\int \qquad dx$

2. (20 pts) Determine whether or not the following improper integrals converge or diverge.
(a) If divergent: say so and prove/explain.
(b) If convergent: say so and prove/explain AND, if possible, find the value of the integral.
(c) Please explain!!! No explanation ⇒ NO POINTS!
[Hint: If you cannot evaluate the integral consider using the comparison (sandwich) test].

a) (5 pts) $I_3 = \displaystyle\int_{-\infty}^{2} 4\,e^{2x}\,dx.$

I_3 :

b) (5 pts) $I_4 = \displaystyle\int_{1}^{\infty} \frac{x}{2x^3 + 4\,\cos^2(x)}\,dx.$

I_4 :

c) (5 pts) $I_5 = \displaystyle\int_{3}^{4} \frac{2}{x-3}\,dx.$

I_5 :

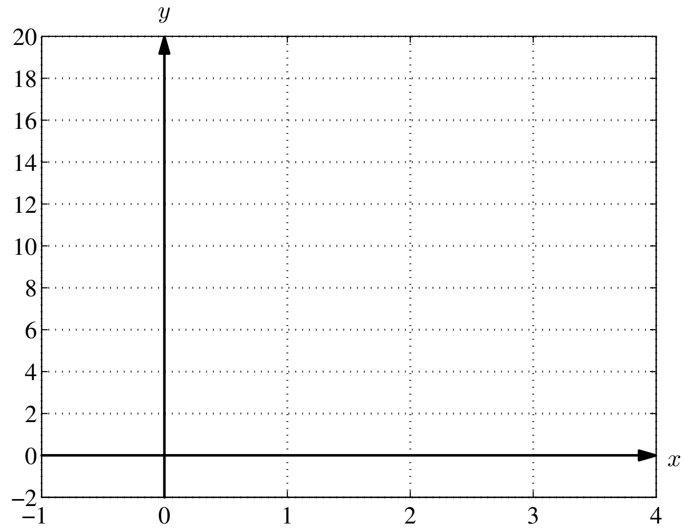
d) (5 pts) $I_6 = \displaystyle\int_{3}^{\infty} \left(\frac{4}{x} + 3\,e^{-x}\right)dx.$

I_6 :

3. (15 pts) Write an explicit integral giving the length of the curve defined by the graph of $\boldsymbol{y = f(x) = x^2}$ for $\boldsymbol{1 \leq x \leq 4}$ using **(a)** an integral over x and **(b)** an integral over y. **You do NOT need to compute these integrals.**
(c) Draw a sketch **including the coordinates of the initial and final points**!

(a) Using integral over x:

(c)



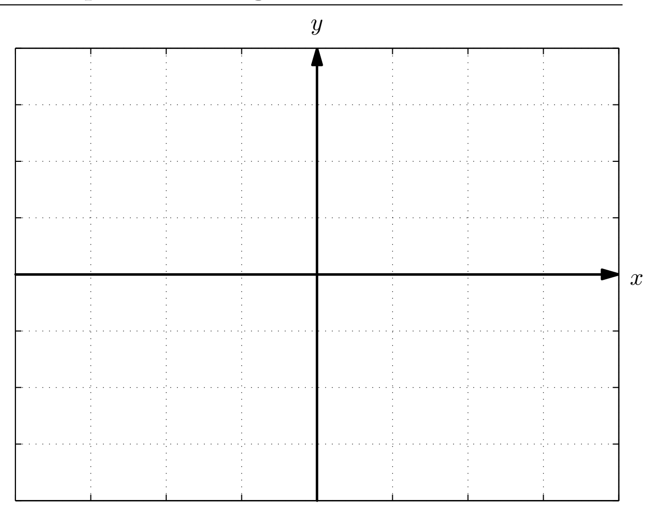
$L_x = \int_{\square}^{\square} \square \; d\square$

(b) Using integral over y:

$L_y = \int_{\square}^{\square} \square \; d\square$

4. (10 pts) An official American football's profile can be approximated by rotating about the x-axis the positive part of the parabola: $\boldsymbol{y = 9 - \frac{1}{25}x^2}$ where x and y are measured in cm. [I am not making this up, this is a very close approximation to a regular American football !]
(a) Draw a sketch. Find the vertex and the roots of the parabola and plot them!
(b) Write an integral for the football's surface area by rotating the parabola about the x-axis. Do not forget the intervals of integration. **You do NOT need to compute the integral!**

(a)



Roots: , ; Vertex: (,)

(b)

$S_1 = \int_{\square}^{\square} \square \; d\square$

5. **(5 pts)** For the differential equation:

$$\boldsymbol{y' = y - \frac{x}{2}},$$

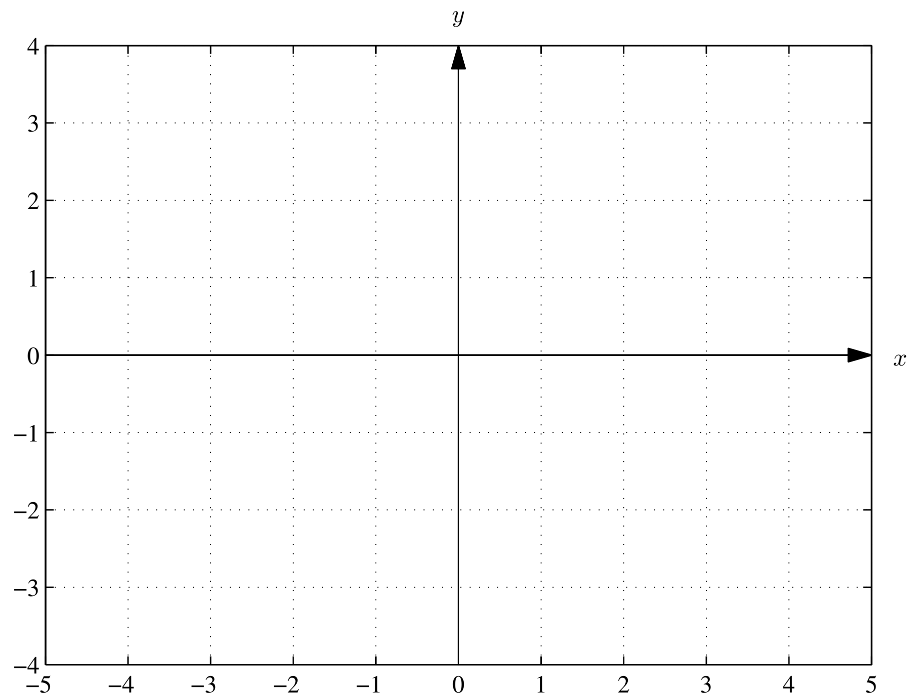
Sketch:

(a) its direction field,

(b) a few (8 or so) of its solutions (using thin lines),

(c) the solution with initial condition: $\boldsymbol{y(3) = 1}$ (using a **thick** line), and

(d) plot the locatin of the initial condition using a LARGE dot.



6. **(10 pts)** For the family of curves $\mathcal{F}$: $\boldsymbol{y = B/x}$ where $\boldsymbol{B}$ is an arbitrary constant:

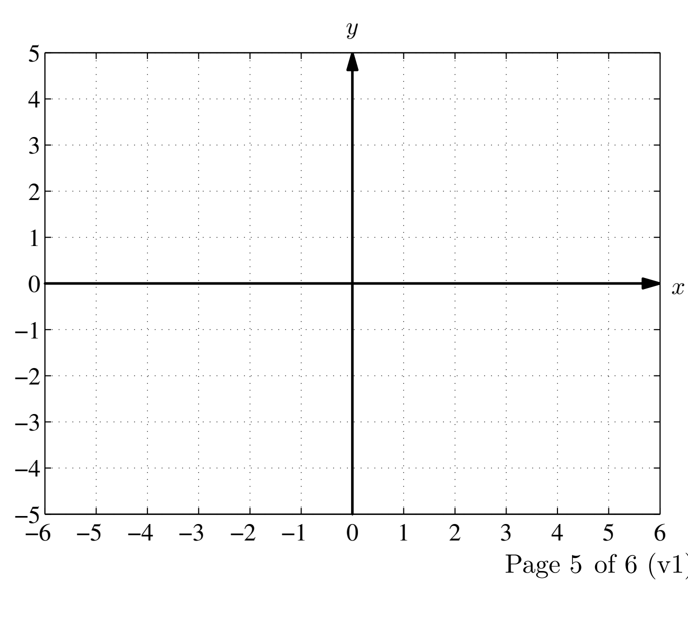
(a) Use DIFFERENTIAL EQUATIONS to find the orthogonal curves to this family.

(b) Plot a sketch of the two families together (use solid for original family $\mathcal{F}$ and dashed for the orthogonal family).

(a)

Orthogonal family: $y(x) =$

(b)



7. (10 pts) Solve the following differential equation satisfying the given initial conditions.
(a) Give first the general solution and then **(b)** the particular solution satisfying the initial condition.

$$y' - \frac{\cos(x)}{y} = 0 \text{ with } y(\pi) = 5.$$

(a) General sol: $y(x) =$

(b) Particular sol: $y(x) =$

8. (5 pts) Find the general solution to the following differential equation: $x^2\, y' + 3\, x\, y = 4$.

[extra (+2 pts)]: Verify that the solution you found does indeed solve the differential equation: