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Midterm #2 (v1) — Math 151 — Calculus II — Fall 2018

I, ____, student of section ________, pledge that this material is completely my own work, and that I did not take, borrow, or copy any portions from any other person(s). I understand if I violate this honesty pledge, I am subject to disciplinary actions pursuant to the appropriate sections of the San Diego State University Policies.

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Signature

(0) Write your first and last name above using CAPITAL LETTERS.
(1) If you use pencil please **use pressure!!!**
If you write softly with pencil the scan will be unreadable and your test will NOT be graded!
(2) Do NOT alter the QR-code above! If you do so, your paper will not be graded and you will get a ZERO.
(3) Do NOT open this test booklet until told to do so.
(4) Do ALL your work on this test booklet.
(5) If you need extra space please use the last 1.5 pages.
(6) NO CALCULATORS, NO CHEAT-SHEETS or any other aids allowed.
(7) You may write in either pen or pencil, but answers deemed illegible will be ignored.
(8) Please enter your answers in the BOXES provided
(9) Please check that all **8 pages** (including this cover sheet) are intact.
(10) The value for each question is given in the table below.
(11) In all the questions you should indicate how you arrived at your answer.
(12) To get full credit you need to simplify your answers (cf. $\sin(0) = 0, e^0 = 1, \sqrt{4} = 2, 2/4 = 1/2$, etc...).

1	2	3	4	5	6	7	8	9	Total
/7	/8	/12	/10	/15	/8	/10	/10	/5	/85

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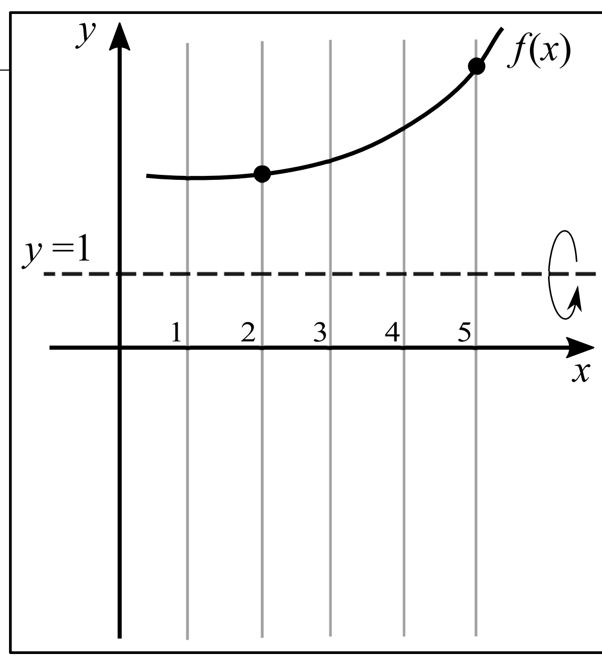
1. (7 pts) Integrate:

$$\boldsymbol{I_1 = \int \frac{5x-1}{x^2-x-2}\,dx =}$$

$I_1 =$

2. (8 pts) Write an explicit integral for the SURFACE AREA obtained by rotating about the line $\boldsymbol{y = 1}$ (note this is not one of the main axes!) the graph of $\boldsymbol{y = f(x)}$ for $\boldsymbol{2 \le x \le 5}$.
DRAW A SKETCH. **Do NOT compute this integral!**

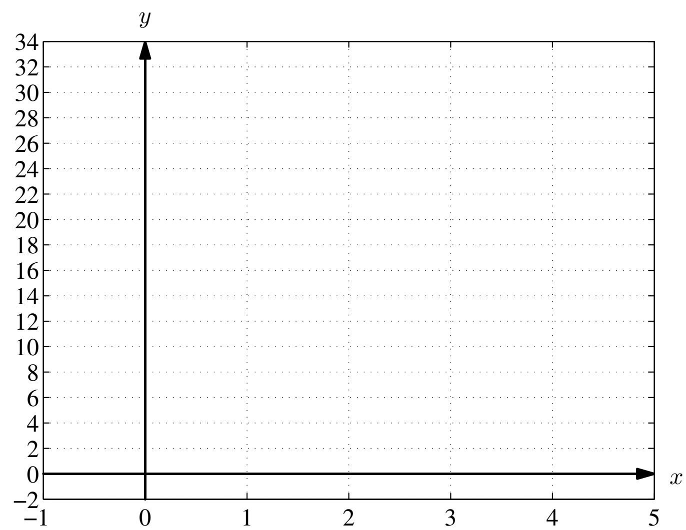


$S = \int_{\square}^{\square} \square \; d\square$

3. (12 pts) Write an explicit integral giving the length of the curve defined by the graph of $\boldsymbol{y = f(x) = 3\,x^2}$ for $\boldsymbol{0 \le x \le 3}$ using **(a)** an integral over x and **(b)** an integral over y. **You do NOT need to compute these integrals.**
(c) Draw a sketch **including the locations of the initial and final points**!

(a) Using integral over x:

(c)



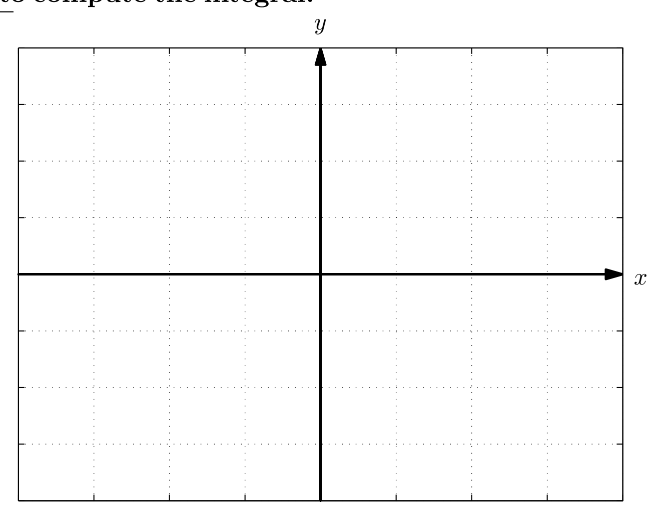
$L_x = \int_{\square}^{\square} \square \, d\square$

(b) Using integral over y:

$L_y = \int_{\square}^{\square} \square \, d\square$

4. (10 pts) An official American football's profile can be approximated by rotating about the x-axis the positive part of the parabola: $\boldsymbol{y = 9 - \frac{1}{25}x^2}$ where x and y are measured in cm.
[I am not making this up, this is a very close approximation to a regular American football !]
(a) Draw a sketch. Find the vertex and the roots of the parabola and plot them!
(b) Write an integral for the football's surface area by rotating the parabola about the x-axis. Do not forget the intervals of integration. **You do NOT need to compute the integral!**

(a)



Roots: ______ , ______ ; Vertex: (______ , ______)

(b)

$S_1 = \int_{\square}^{\square} \square \, d\square$

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5. **(15 pts)** Determine whether or not the following improper integrals converge or diverge.

(i) If divergent: say so and prove/explain.

(ii) If convergent: say so and prove/explain AND, if possible, find the value of the integral.

(iii) Please explain!!! No explanation ⇒ NO POINTS!

[Hint: If you cannot evaluate the integral consider using the comparison (sandwich) test].

a) **(5 pts)** $\boldsymbol{I_2 = \int_{-\infty}^{3} 4\, e^{2x}\, dx}$.

I_2 :

b) **(5 pts)** $\boldsymbol{I_3 = \int_{1}^{\infty} \frac{1}{5x^2 + 4\, e^{-x}}\, dx}$.

I_3 :

c) **(5 pts)** $\boldsymbol{I_4 = \int_{3}^{4} \frac{2}{x-3}\, dx}$.

I_4 :

6. (8 pts) Solve the following differential equation satisfying the given initial conditions.
(a) Give first the general solution and then **(b)** the particular solution satisfying the initial condition.

$$\boldsymbol{y' - \frac{\cos(x)}{y} = 0} \text{ with } \boldsymbol{y(\pi) = 2}.$$

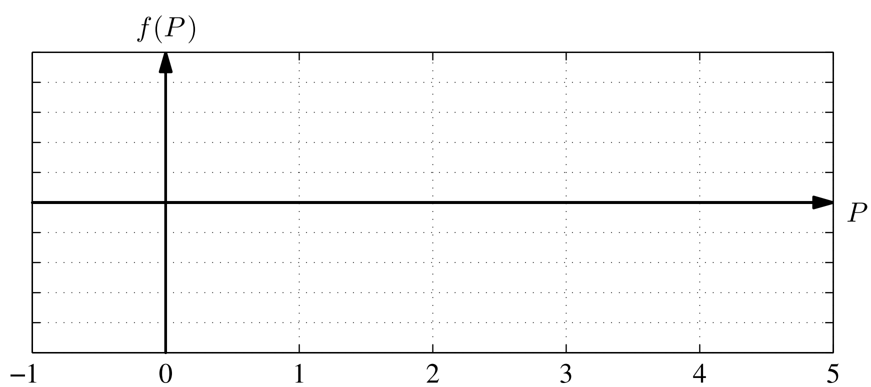
(a) General sol: $y(x) =$

(b) Particular sol: $y(x) =$

7. (10 pts) A population $P(t)$ behaves according to the differential equation: $\boldsymbol{\frac{dP}{dt} = f(P) = P\,(P-2)\,(P-4)}$.
Perform the following tasks:

(a) (i) Draw a sketch for $f(P)$ as a function of P. [You do not need to tabulate the function! Just use the roots (and the limits at $P \to \pm\infty$) to draw a rough sketch!]
(ii) Find the the roots of f and PLOT THEM.
(iii) Include arrows on the P-axis indicating the direction of the flow.

Roots:



(b) Give the intervals where the population is increasing/decreasing. **Use standard set notation: (·), [·], [·), ∪, ...**

P is increasing on:

P is decreasing on:

(c) For the following initial population $P(0) = P_0$ indicate where will the population settle after long times:

If $P_0 = 0$ then $P(t)$ settles/goes to:

If $P_0 = 0.5$ then $P(t)$ settles/goes to:

If $P_0 = 2$ then $P(t)$ settles/goes to:

If $P_0 = 3.5$ then $P(t)$ settles/goes to:

If $P_0 = 4$ then $P(t)$ settles/goes to:

If $P_0 = 4.5$ then $P(t)$ settles/goes to:

8. (10 pts) Mixing problem: Dialysis treatment removes urea (or other waste products) from a patient's body by diverting some blood flow to a filter that completely filters out the urea such that a clean blood flow (without urea) is channelled back into the patient while the patient's total blood volume is kept constant. An average individual has **5 liters** of blood and a standard dialysis flow is **2 liters per minute**. Suppose that the patient starts the dialysis treatment with a concentration of urea of **3 grams/liter**.

(a) Write a differential equation & its initial condition [i.e. the total amount of urea (in grams) in the patient at the beginning of the dialysis] for the **TOTAL AMOUNT OF UREA** in the patient $y(t)$ (in grams) as a function of time t (in minutes).

DE: , IC:

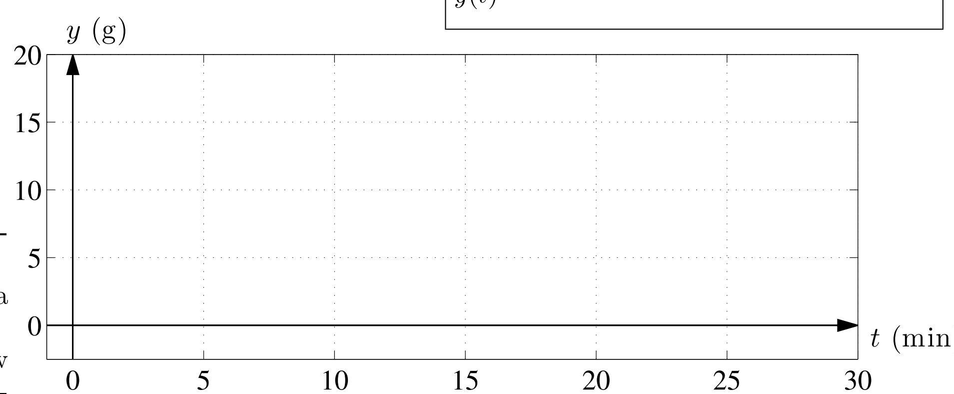
(b) Find the general solution to this differential equation.

$y(t) =$

(c) Find the particular solution satisfying the initial condition.

$y(t) =$

(d) **(i)** Draw a sketch for the direction field of the differential equation you found in (a) and **(ii)** include a sketch (in bold) of the solution you obtained in (c) above.



(e) **[extra credit]**

(i) What is the amount of urea (in grams) left inside the patient after 1/2 hour of dialysis?

(ii) How long would it take for the amount of urea to be half of the amount before treatment?

(iii) According to the solution found in **(c)**, how much urea will be left in the patient if the dialysis continues forever? Explain!

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9. (5 pts) Find the general solution for the following differential equation,

$\boldsymbol{y' + a\,y = b\,e^{d\,x}}$ where $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{d}$ are CONSTANTS.

$y(x) =$

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