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Midterm #2 (v1) — Math 151 — Calculus II — Fall 2019

I, _____, student of section _____, pledge that this material is completely my own work, and that I did not take, borrow, or copy any portions from any other person(s). I understand if I violate this honesty pledge, I am subject to disciplinary actions pursuant to the appropriate sections of the San Diego State University Policies.

_____

Signature

(0) Write your first and last name above using **LARGE CAPITAL LETTERS**: 
(1) If you use pencil please **use pressure!!!**
If you write softly with pencil the scan will be unreadable and your test will NOT be graded!
(2) Do NOT alter the QR-code above! If you do so, your paper will not be graded and you will get a ZERO.
(3) Do NOT open this test booklet until told to do so.
(4) Do ALL your work on this test booklet.
(5) If you need extra space please use the last page.
(6) NO CALCULATORS, NO CHEAT-SHEETS or any other aids allowed.
(7) You may write in either pen or pencil, but answers deemed illegible will be ignored. (see point#1 above)
(8) Please enter your answers in the BOXES provided
(9) Please check that all **8 pages** (including this cover sheet and the extra space page at the end) are intact.
(10) The value for each question is given in the table below.
(11) In all the questions you should indicate how you arrived at your answer.
(12) To get full credit you need to simplify your answers (cf. $\sin(0) = 0, e^0 = 1, \sqrt{4} = 2, 2/4 = 1/2$, etc...).

1	2	3	4	5	6	7	8	9	10	Total
/10	/8	/8	/8	/8	/10	/6	/8	/8	/8	/82

1. (10 pts) Integrate:

a) (7 pts) $\boldsymbol{I_1 = \int \frac{x+2}{x^3 - 2x^2 + x}\, dx =}$

$I_1 =$

b) (3 pts) Write the partial fraction decomposition for the following integral. Do NOT compute the coefficients of the numerators but you MUST JUSTIFY each term in your decomposition (i.e., repeated/non-repeated, linear, quadratic, ...).

Note: you might NOT need to use all the boxes!

$$I_2 = \int \frac{2x^4 - 3x^2 - 9x + 3}{x^2\,(x^2+5)\,(3x-7)\,(x^2+2)^2}\, dx$$

Justification for EACH term:

$$I_2 = \int \square + \square + \square + \square + \square + \square + \square\, dx$$

2. **(8pts)** Write an explicit integral giving the length of the curve defined by the graph of $\boldsymbol{y = f(x) = 3 + 2\cos x}$ for $\boldsymbol{0 \le x \le 2\pi}$ using **(a)** an integral over x and **(b)** an integral over y. **You do NOT need to compute these integrals.** **(c)** Draw a sketch **including the locations of the initial and final points**!

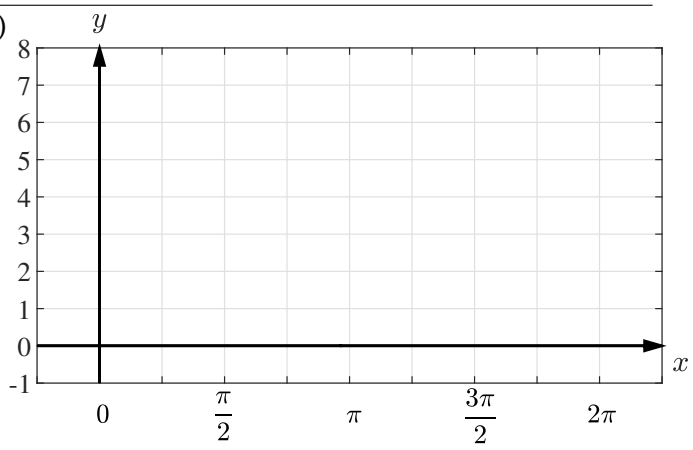
(a) Using integral over x:

$L_x = \int_{\square}^{\square} \square \; d\square$

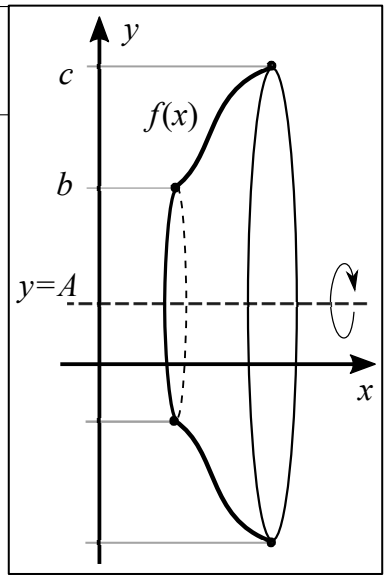
(b) Using integral over y:

$L_y =$

(c)



3. **(8 pts)** Write BOTH an x and a y integral for the SURFACE AREA obtained by rotating about the line $\boldsymbol{y = +A}$ (where $A > 0$) the function $f(x)$ as depicted on the plot to the right. Note that rotation is **NOT** about the x-axis!



$S_x = \int_{\square}^{\square} \square \; dx$

$S_y = \int_{\square}^{\square} \square \; dy$

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4. (8 pts) Show using the methods learned in class that the surface area of the cone of circular base of radius R and height H is $A = \pi R\sqrt{H^2 + R^2}$ **(do NOT include the area of the base)**. Clearly indicate which method you are using, the function(s) that you are plotting, and the interval of integration. Please use a graph to show these properties.

5. (8 pts) (a) Determine whether the following improper integral converges or diverges using the **comparison theorem**. (b) If convergent give an **upper bound** for its value. **Please explain in detail !!!**

$$\boldsymbol{I_3 = \int_1^{\infty} \frac{3 + 2e^{-4x}}{x^2}\, dx}$$

(a) Convergence for I_3 :
(b) Upper bound for I_3 :

6. (10 pts) Determine whether or not the following improper integrals converge or diverge.
(i) If divergent: say so and prove/explain.
(ii) If convergent: say so and prove/explain AND, if possible, find the value of the integral.
(iii) Please explain!!! No explanation ⇒ NO POINTS!

a) (5 pts) $\boldsymbol{I_4 = \int_2^\infty \left(5\,e^{-y/4} + \frac{5}{y^2}\right) dy.}$

I_4 :

b) (5 pts) $\boldsymbol{I_5 = \int_2^\infty \frac{5}{(x-2)^2}\, dx.}$

I_5 :

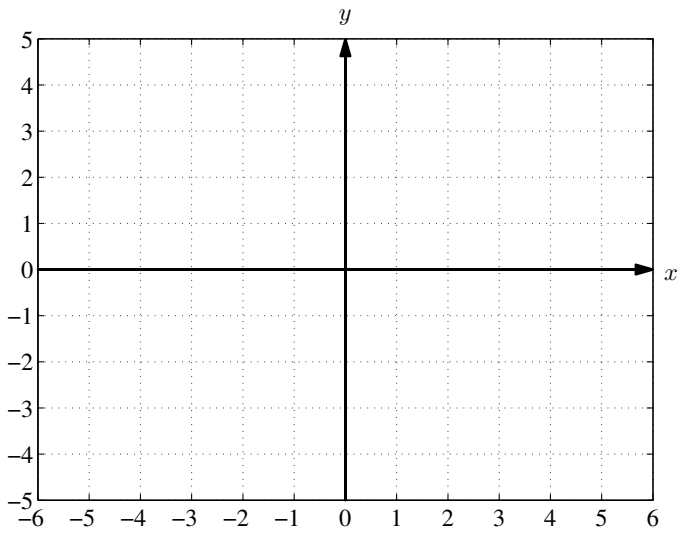
7. (6 pts) For the family of curves $\mathcal{F}$: $\boldsymbol{y = 3\,x + K}$ where K is an arbitrary constant:
(a) Use DIFFERENTIAL EQUATIONS to find the orthogonal curves to this family.
(b) What geometrical objects do the original family $\mathcal{F}$ and the orthogonal family represent? Be SPECIFIC!
(c) Plot a sketch of the two families together (use solid for original family $\mathcal{F}$ and dashed for the orthogonal family).

(a)

($\mathbf{b}_1$) Original family:

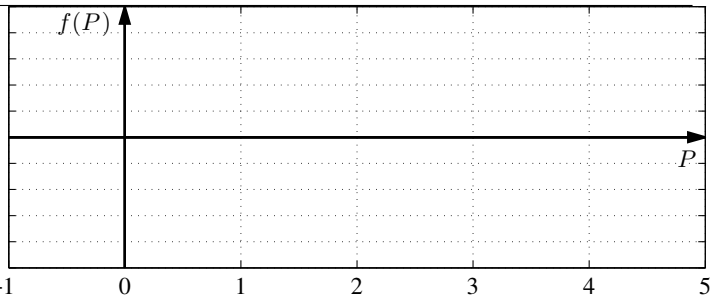
($\mathbf{b}_2$) Orthogonal family:

(c)



8. (8 pts) A population $P(t)$ behaves according to the differential equation: $\dfrac{\boldsymbol{dP}}{\boldsymbol{dt}} = \boldsymbol{f(P) = P\,(P-2)\,(P-4)}$.
Consider that P can take NEGATIVE values. Perform the following tasks:

(a) (i) Draw a sketch for $f(P)$ as a function of P. [You do not need to tabulate the function! Just use the roots (and the limits at $P \to \pm\infty$) to draw a rough sketch!]
(ii) Find the the roots of f and PLOT THEM.
(iii) Include arrows on the P-axis indicating the direction of the flow.



Roots:

(b) Give the intervals where the population is increasing/decreasing. **Use standard set notation: $(\cdot)$, $[\cdot]$, $[\cdot)$, $\cup$, ...**

P is increasing on:

P is decreasing on:

(c) For the following initial population $P(0) = P_0$ indicate where will the population settle after long times:

If $P_0 = 0$ then $P(t)$ settles/goes to:

If $P_0 = 1$ then $P(t)$ settles/goes to:

If $P_0 = 2$ then $P(t)$ settles/goes to:

If $P_0 = 3$ then $P(t)$ settles/goes to:

If $P_0 = 4$ then $P(t)$ settles/goes to:

If $P_0 = 5$ then $P(t)$ settles/goes to:

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9. (8 pts) Solve the following differential equation satisfying the given initial conditions.
(a) Give first the general solution and then **(b)** the particular solution satisfying the initial condition.

$\boldsymbol{y\,y' - \cos(x) = 0}$ with $\boldsymbol{y(\pi) = 2}$.

(a) General sol: $y(x) =$

(b) Particular sol: $y(x) =$

10. (8 pts) Compute the following integral using **TRIGONOMETRIC SUBSTIUTION**. No u-sub allowed!
NOTE: Do NOT, I say NOT, leave your result in terms of trig. functions.

$$\boldsymbol{I_6 = \int \frac{x}{\sqrt{5 - 2x^2}}\,dx}$$

$I_6 =$

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This blank page should be used as scratch paper.