Midterm #2 (v1) — Math 151 — Calculus II — Spring 2018

Professor/TA: ____________________ **Sec:** ________ **RedID:** ____________

NAME (printed): ____________ ____________
(Last Name) **(First Name)**

I, ______________________, pledge that this material is completely my own work, and that I did not take, borrow, or copy any portions from any other person(s). I understand if I violate this honesty pledge, I am subject to disciplinary actions pursuant to the appropriate sections of the San Diego State University Policies.

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Signature

(1) Do NOT open this test booklet until told to do so.
(2) Do ALL your work on this test booklet.
(3) If you need extra space please ask instructor for extra paper.
(4) NO CALCULATORS, NO CHEAT-SHEETS or any other aids allowed.
(5) You may write in either pen or pencil, but answers deemed illegible will be ignored.
(6) Please enter your answers in the BOXES provided
(7) Please check that all **6 pages** (including this cover sheet) are intact.
(8) The value for each question is given in the table below.
(9) In all the questions you should indicate how you arrived at your answer.
(10) To get full credit you need to simplify your answers (cf. $\sin(0) = 0, e^0 = 1, \sqrt{4} = 2, 2/4 = 1/2$, etc...).

1	2	3	4	5	6	7	8	Total
/12	/8	/20	/12	/8	/10	/10	/15	/95

1. (12 pts) Integrals.

a) (7 pts) $I_1 = \displaystyle\int \frac{x+3}{x^2+x-2}\, dx =$

$I_1 =$

b) (5 pts) Write the partial fraction decomposition for the following integral. Do NOT compute the coefficients of the numerators but you MUST JUSTIFY each term in your decomposition (i.e., repeated/non-repeated, linear, quadratic, ...).

Note: you might NOT need to use all the boxes!

$$I_2 = \int \frac{3x^4 - 4x^2 - 2x + 1}{x^2\ (x^2+5)\ (x^2+3)^2\ (2x-3)}\, dx$$

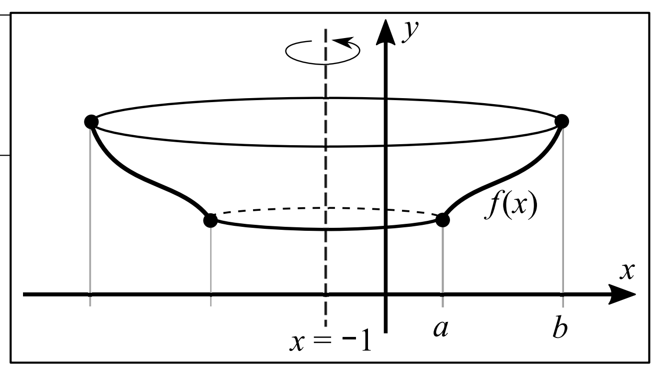
Justification for EACH term:

$$\boxed{} \quad \boxed{} \quad \boxed{} \quad \boxed{} \quad \boxed{} \quad \boxed{} \quad \boxed{}$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$I_2 = \int \boxed{} + \boxed{} + \boxed{} + \boxed{} + \boxed{} + \boxed{} + \boxed{}\ dx$$

2. (8 pts) Write BOTH an x and a y integral for the SURFACE AREA obtained by rotating about the line $\boldsymbol{x = -1}$ the function $f(x)$ as depicted on the plot to the right. Note that rotation is NOT about the y-axis!



$$S_x = \int_{\boxed{}}^{\boxed{}} \boxed{}\ dx$$

$$S_y = \int_{\boxed{}}^{\boxed{}} \boxed{}\ dy$$

3. (20 pts) Determine whether or not the following improper integrals converge or diverge.
(i) If divergent: say so and prove/explain.
(ii) If convergent: say so and prove/explain AND, if possible, find the value of the integral.
(iii) Please explain!!! No explanation $\Rightarrow$ NO POINTS!
[Hint: If you cannot evaluate the integral consider using the comparison (sandwich) test].

a) (5 pts) $I_3 = \displaystyle\int_0^\infty 3\,e^{-2x}\,dx.$

I_3 :

b) (5 pts) $I_4 = \displaystyle\int_1^\infty \frac{2}{x^2+3\sqrt{x}}dx.$

I_4 :

c) (5 pts) $I_5 = \displaystyle\int_3^6 \frac{2}{x-3}\,dx.$

I_5 :

d) (5 pts) $I_6 = \displaystyle\int_3^\infty \left(\frac{1}{x^2} + 3\,e^{-x}\right)\,dx.$

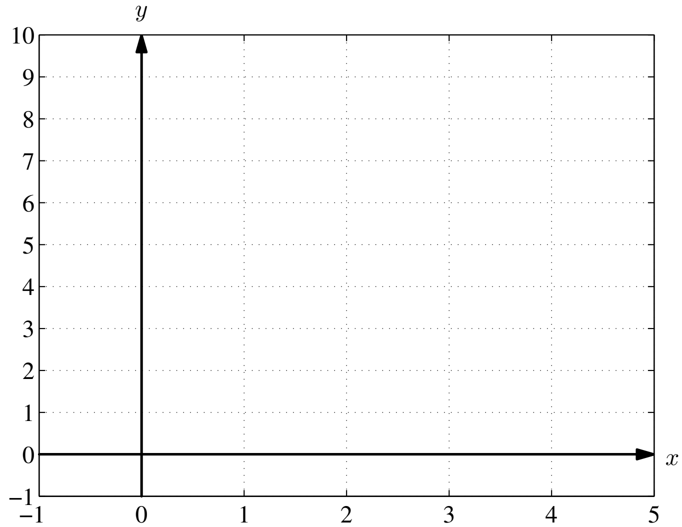
I_6 :

4. **(12 pts)** Write an explicit integral giving the length of the curve defined by the graph of $\boldsymbol{y = f(x) = 3\sqrt{x}}$ for $\boldsymbol{1 \le x \le 4}$ using **(a)** an integral over x and **(b)** an integral over y. **You do NOT need to compute these integrals.**
(c) Draw a sketch **including the coordinates of the initial and final points**!

(a) Using integral over x:

$$L_x = \int_{\square}^{\square} \boxed{} \, d\square$$

(c)



(b) Using integral over y:

$$L_y = \int_{\square}^{\square} \boxed{} \, d\square$$

5. **(8 pts)** Show using the methods learned in class that the surface area of the cone of circular base of radius R and height H is $A = \pi R\sqrt{H^2 + R^2}$ **(do NOT include the area of the base)**. Clearly indicate which method you are using, the function(s) that you are plotting, and the interval of integration. Please use a graph to show these properties.

6. (10 pts) Solve the following differential equation satisfying the given initial conditions.
(a) Give first the general solution and then **(b)** the particular solution satisfying the initial condition.

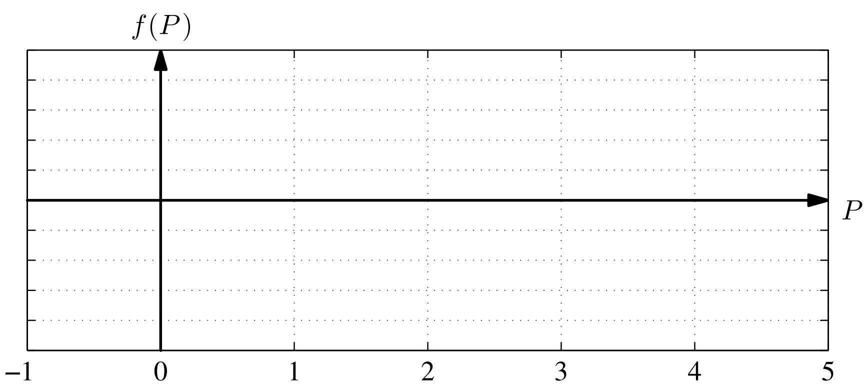
$$\boldsymbol{y' - \frac{7x^6}{8y} = 0 \text{ with } y(0) = -1.}$$

(a) General sol: $y(x) =$

(b) Particular sol: $y(x) =$

7. (10 pts) A population $P(t)$ behaves according to the differential equation: $\dfrac{dP}{dt} = f(P) = P\,(P-2)\,(P-4)$.
Perform the following tasks:

(a) (i) Draw a sketch for $f(P)$ as a function of P. [You do not need to tabulate the function! Just use the roots (and the limits at $x \to \pm\infty$) to draw a rough sketch!]
(ii) Find and plot in the sketch the roots of f.
(iii) Include arrows on the x-axis indicating the direction of the flow.



Roots:

(b) Give the population intervals where the population is increasing/decreasing. Use standard set notation: $(\cdot)$, $[\cdot]$, $[\cdot)$, $\cup$, ...

P is increasing on:

P is decreasing on:

(c) For the following initial population $P(0) = P_0$ indicate where will the population settle after long times:

If $P_0 = 0$ then $P(t)$ settles/goes to:	If $P_0 = 0.5$ then $P(t)$ settles/goes to:
If $P_0 = 2$ then $P(t)$ settles/goes to:	If $P_0 = 3.5$ then $P(t)$ settles/goes to:
If $P_0 = 4$ then $P(t)$ settles/goes to:	If $P_0 = 4.5$ then $P(t)$ settles/goes to:

8. (15 pts) TRIG. SUB.: **NOTE: you cannot leave your result as a composition of an inverse trig. function inside a trig. function (or vice-versa). A single inverse trig. is ok.** Hints: $\tan' = \sec^2$, $1 + \tan^2 = \sec^2$

a) (10 pts) Evaluate the following integral using **TRIGONOMETRIC SUBSTITUTION**.
Yes, I know, this integral can also be done by u-sub. However, do it using TRIG SUB!

$$\boldsymbol{I_7 = \int \frac{x}{\sqrt{16 - x^2}}\,dx}$$

$I_7 =$

b) (5 pts) Using trig. substitution, write the following integral as a trigonometric integral, i.e. rewrite it as $I_8 = \int_{\theta_1}^{\theta_2} f(\theta)\,d\theta$.
Notes: Clearly state the trigonometric substitution. Do NOT evaluate the integral, just write it as $I_8 = \int_{\theta_1}^{\theta_2} f(\theta)\,d\theta$.

$$\boldsymbol{I_8 = \int_{-2}^{1} \sqrt{x^2 + 4x + 13}\,dx}$$

$I_8 = \int_{\square}^{\square} \square \, d\theta$