Do NOT write ANYTHING above this line!

Midterm #2 (v1) — Math 151 — Calculus II — Spring 2019

I, ____, student of section ________, pledge that this material is completely my own work, and that I did not take, borrow, or copy any portions from any other person(s). I understand if I violate this honesty pledge, I am subject to disciplinary actions pursuant to the appropriate sections of the San Diego State University Policies.

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Signature

(0) Write your first and last name above using **LARGE CAPITAL LETTERS**.
(1) If you use pencil please **use pressure!!!**
If you write softly with pencil the scan will be unreadable and your test will NOT be graded!
(2) Do NOT alter the QR-code above! If you do so, your paper will not be graded and you will get a ZERO.
(3) Do NOT open this test booklet until told to do so.
(4) Do ALL your work on this test booklet.
(5) If you need extra space please use the last HALF page.
(6) NO CALCULATORS, NO CHEAT-SHEETS or any other aids allowed.
(7) You may write in either pen or pencil, but answers deemed illegible will be ignored. (see point#1 above)
(8) Please enter your answers in the BOXES provided
(9) Please check that all **8 pages** (including this cover sheet and the extra space page at the end) are intact.
(10) The value for each question is given in the table below.
(11) In all the questions you should indicate how you arrived at your answer.
(12) To get full credit you need to simplify your answers (cf. $\sin(0) = 0, e^0 = 1, \sqrt{4} = 2, 2/4 = 1/2$, etc...).

1	2	3	4	5	6	7	8	9	10	Total
/10	/8	/8	/8	/8	/10	/8	/8	/8	/8	/84

1. (10 pts) Integrate:

a) (2 pts) Using **long division**, prove that $\dfrac{x^3-2x-10}{x^2-x-2} = x+1+\dfrac{x-8}{x^2-x-2}$

b) (5 pts) Using **a)** above, integrate: $I_1 = \displaystyle\int \frac{x^3-2x-10}{x^2-x-2}\,dx =$

$I_1 =$

c) (3 pts) Write the partial fraction decomposition for the following integral. Do NOT compute the coefficients of the numerators but you MUST JUSTIFY each term in your decomposition (i.e., repeated/non-repeated, linear, quadratic, ...).

Note: you might NOT need to use all the boxes!

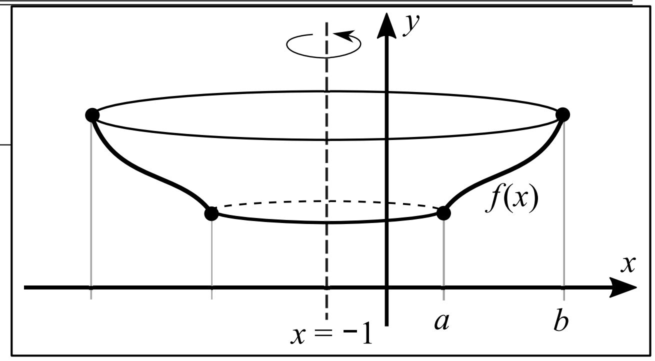
$$I_2 = \int \frac{2x^4-5x^2-2x+5}{x^2\,(x^2+4)\,(x^2+3)^2\,(7x-2)}\,dx$$

Justification for EACH term:

$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$

$I_2 = \displaystyle\int \boxed{} + \boxed{} + \boxed{} + \boxed{} + \boxed{} + \boxed{} + \boxed{}\,dx$

2. (8 pts) Write BOTH an x and a y integral for the SURFACE AREA obtained by rotating about the line $\boldsymbol{x = -1}$ the function $f(x)$ as depicted on the plot to the right.
Note that rotation is NOT about the y-axis!



$$S_x = \int_{\square}^{\square} \boxed{} \, dx$$

$$S_y = \int_{\square}^{\square} \boxed{} \, dy$$

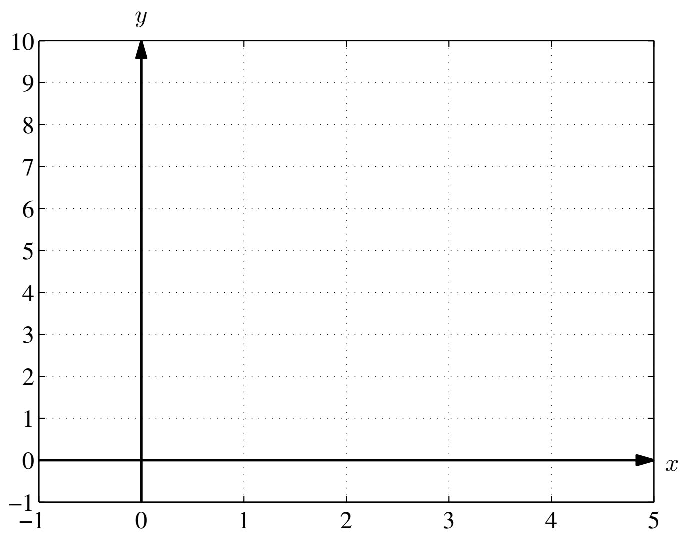
3. (8pts) Write an explicit integral giving the length of the curve defined by the graph of $\boldsymbol{y = f(x) = 3\sqrt{x}}$ for $\boldsymbol{1 \leq x \leq 4}$ using **(a)** an integral over x and **(b)** an integral over y. **You do NOT need to compute these integrals.**
(c) Draw a sketch **including the locations of the initial and final points**!

(a) Using integral over x:

$$L_x = \int_{\square}^{\square} \boxed{} \, d\square$$

(b) Using integral over y:

(c)



$$L_y = \int_{\square}^{\square} \boxed{} \, d\square$$

4. (8 pts) Show, using surfaces of revolution, that the surface of a sphere of radius R is $S = 4\pi\, R^2$.
Clearly indicate which method you are using, the function(s) that you are plotting, and the interval of integration.
Please use a graph to show these properties.

5. (8 pts) (a) Determine whether the following improper integral converges or diverges using the **comparison theorem**.
(b) If convergent give an upper bound for its value.
Please explain in detail!!! [Hint: $-1 \le \cos(x) \le 1$].

$$I_3 = \int_{\pi}^{\infty} \frac{3 + \cos^2(x)}{x^2}\, dx$$

(a) Convergence for I_3 :

(b) Upper bound for I_3 :

Do NOT write ANYTHING above this line!

6. **(10 pts)** Determine whether or not the following improper integrals converge or diverge.
(i) If divergent: say so and prove/explain.
(ii) If convergent: say so and prove/explain AND, if possible, find the value of the integral.
(iii) Please explain!!! No explanation ⇒ NO POINTS!

a) **(5 pts)** $I_4 = \displaystyle\int_0^{\infty} 3\,e^{-2x}\,dx.$

I_4 :

b) **(5 pts)** $I_5 = \displaystyle\int_2^{5} \frac{2}{(x-2)^2}\,dx.$

I_5 :

7. **(8 pts)** Solve the following differential equation satisfying the given initial conditions.
(a) Give first the general solution and then **(b)** the particular solution satisfying the initial condition.

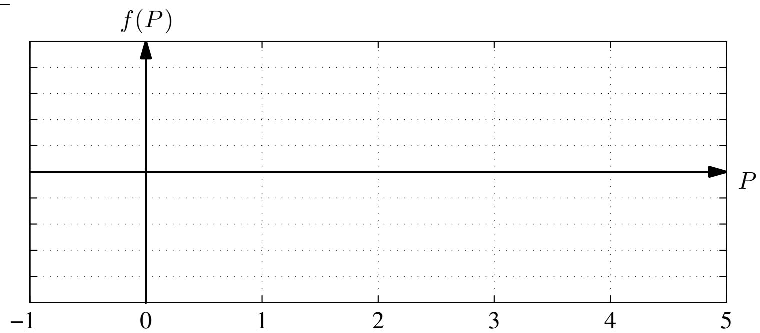
$\boldsymbol{y' - A\,x^b\,y^2 = 0}$ with $\boldsymbol{y(0) = 3}$ where $\boldsymbol{A}$ and $\boldsymbol{b}$ are fixed constants.

(a) General sol: $y(x) =$

(b) Particular sol: $y(x) =$

8. **(8 pts)** A population $P(t)$ behaves according to the differential equation: $\dfrac{\boldsymbol{dP}}{\boldsymbol{dt}} = \boldsymbol{f(P) = (P-1)\,(P-2)\,(P-3)}$.
Perform the following tasks:

(a) **(i)** Draw a sketch for $f(P)$ as a function of P.
[You do not need to tabulate the function! Just use the roots (and the limits at $P \to \pm\infty$) to draw a rough sketch!]
(ii) Find the the roots of f and PLOT THEM.
(iii) Include arrows on the P-axis indicating the direction of the flow.



Roots:

(b) Give the intervals where the population is increasing/decreasing. **Use standard set notation:** $(\cdot)$, $[\cdot]$, $[\cdot)$, $\cup$, ...

P is increasing on:

P is decreasing on:

(c) For the following initial population $P(0) = P_0$ indicate where will the population settle after long times:

If $P_0 = 0$ then $P(t)$ settles/goes to:

If $P_0 = 1$ then $P(t)$ settles/goes to:

If $P_0 = 1.5$ then $P(t)$ settles/goes to:

If $P_0 = 2$ then $P(t)$ settles/goes to:

If $P_0 = 2.5$ then $P(t)$ settles/goes to:

If $P_0 = 3.5$ then $P(t)$ settles/goes to:

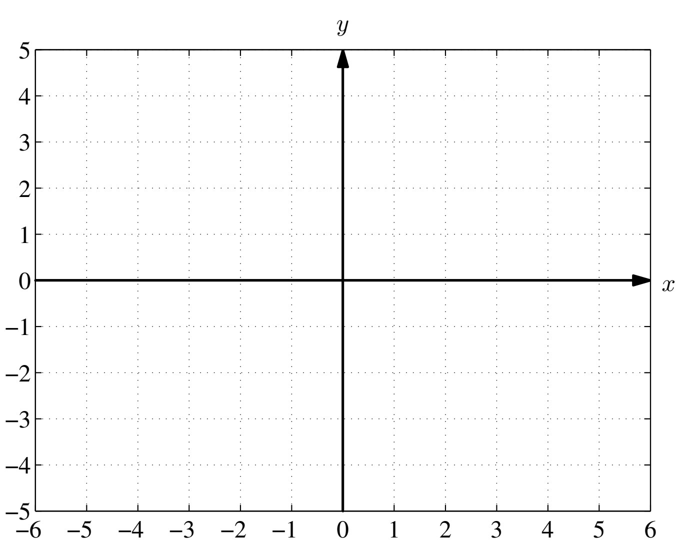
9. **(8 pts)** For the family of curves $\mathcal{F}$: $\boldsymbol{y = A\,x}$ where $\boldsymbol{A}$ is an arbitrary constant:
(a) Use DIFFERENTIAL EQUATIONS to find the orthogonal curves to this family.
(b) What geometrical objects do the original family $\mathcal{F}$ and the orthogonal family represent? Be SPECIFIC!
(c) Plot a sketch of the two families together (use solid for original family $\mathcal{F}$ and dashed for the orthogonal family).

(a)

(b$_1$) Original family:

(b$_2$) Orthogonal family:

(c)



10. (8 pts) A tank with **1000 liters** of water initially contains **4 g** of calcium. Water with a concentration of **2 mg/L** (milligrams/liter) is pumped into the tank at a rate of **1 L/min** and the mixture is pumped out at the same rate.

(a) Write a differential equation and its initial condition for, $\boldsymbol{y(t)}$, the TOTAL amount of calcium **in grams** (not milligrams!) in the tank. ($\boldsymbol{t}$ is measured in minutes. Note: **1g = 1,000 mg**).

Diff. Eq.: , Initial Condition:

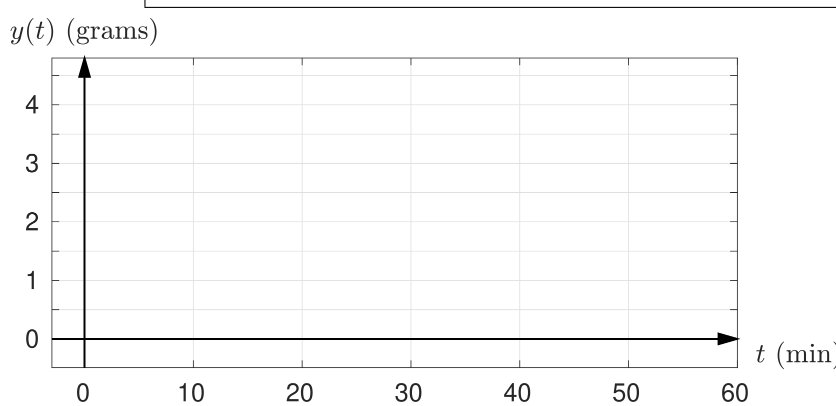
(b) Find the general solution to this differential equation.

$y(t) =$

(c) Find the particular solution satisfying the initial condition.

$y(t) =$

(d) **(i)** Draw a sketch for all possible solutions to this differential equation and **(ii)** include a sketch (in bold) of the solution obtained in **(c)**.



(e) **[extra credit]**

(i) What is the amount of calcium in the tank after 1/2 hour?

(ii) According to the solution found in **(c)**, what is the amount of calcium if we wait forever? Explain! Why?

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