Midterm #3 (v1) — Math 151 — Calculus II — Fall 2017

Professor/TA: ______________________ **Sec:** ________ **RedID:** ______________

NAME (printed): ______________ ______________

(Last Name) **(First Name)**

I, ______________________________, pledge that this material is completely my own work, and that I did not take, borrow, or copy any portions from any other person(s). I understand if I violate this honesty pledge, I am subject to disciplinary actions pursuant to the appropriate sections of the San Diego State University Policies.

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Signature

(1) Do NOT open this test booklet until told to do so.
(2) Do ALL your work on this test booklet.
(3) If you need extra space please ask instructor for extra paper.
(4) NO CALCULATORS, NO CHEAT-SHEETS or any other aids allowed.
(5) You may write in either pen or pencil, but answers deemed illegible will be ignored.
(6) Please enter your answers in the BOXES provided
(7) Please check that all **6 pages** (including this cover sheet) are intact.
(8) The value for each question is given in the table below.
(9) In all the questions you should indicate how you arrived at your answer.
(10) To get full credit you need to simplify your answers (cf. $\sin(0) = 0, e^0 = 1, \sqrt{4} = 2, 2/4 = 1/2$, etc...).

1	2	3	4	5	6	7	8	9	10(xtra)	Total
/5	/10	/10	/5	/20	/10	/ 5	/10	/ 5	/ 5	/80

1. (5 pts) Determine the EXACT value of the following series by expressing it as a telescoping sum.

$$S_1 = \sum_{n=1}^{\infty} \frac{4}{n^2 + n}$$

$S_1 =$

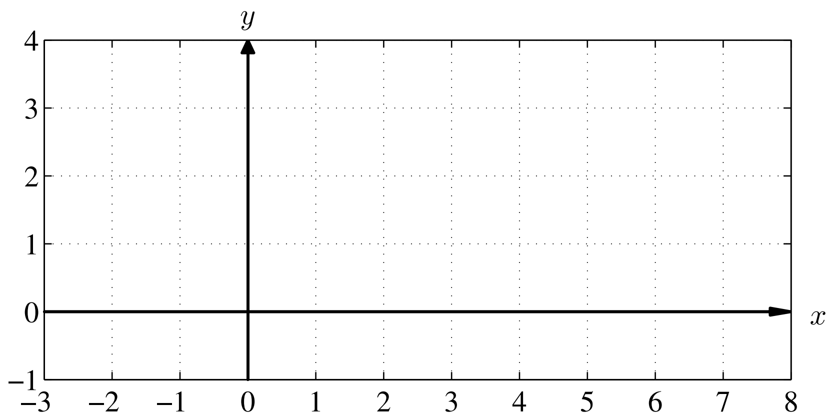
2. (10 pts) Determine the **radius of convergence** AND the **interval of convergence** of the following infinite series. **Do NOT study the convergence at the end points.** Show all your work and explicitly write what test you are using.

$$\boldsymbol{\sum_{n=1}^{\infty} \frac{n^3}{5^n} (3x+2)^n}$$

Radius of convergence:

Interval of convergence:

3. (10 pts) (a) Compute the Taylor series of order 2 (i.e. second degree polynomial) for $\boldsymbol{f(x) = \sqrt{x+2}}$ about $\boldsymbol{x_0 = 2}$. [You do not need to do any convergence]. **(b)** Qualitatively sketch (i) the function (bold curve), its (ii) linear approximation $L(x)$ (thin curve), and (iii) its quadratic approximation $Q(x)$ (dashed curve) and label each curve accordingly.



$f(x) \approx$ ☐ + ☐ + ☐

4. (5 pts) Compute the Taylor series of order 4 (i.e. fourth degree polynomial) for $\boldsymbol{g(x) = A\cos(x)}$ about $\boldsymbol{x_0 = 0}$.

$g(x) \approx$

5. (20 pts) Determine whether the following infinite series converge or diverge USING THE INDICATED TEST. Make sure to check that ALL conditions for each test are satisfied. **No detailed explanations → no points!!!**

a) (5 pts) $\boldsymbol{S_2 = \sum_{n=5}^{\infty} \frac{1}{n-4}}$ (Direct comparison [sandwich] test)

b) (5 pts) $\boldsymbol{S_3 = \sum_{n=1}^{\infty} \frac{2n^2}{4+5n^2}}$ (Divergence test)

c) (5 pts) $\boldsymbol{S_4 = \sum_{n=2}^{\infty} \frac{3\sqrt{n+1}}{n^3-2}}$ (Limit comparison test)

d) (5 pts) $\boldsymbol{S_5 = \sum_{n=1}^{\infty} \frac{2}{\sqrt{2n+1}}}$ (Integral test)

6. (10 pts) Determine whether the following infinite series **converge absolutely**, **converge conditionally** or **diverge**. You must show all your work and **indicate what test you are using**. No detailed explanations → no points!

$$\boldsymbol{S_8 = \sum_{n=0}^{\infty} (-1)^n \frac{n}{n^2+2}}$$

Circle one: diverges converges absolutely converges conditionally

7. (5 pts) Using the fact that the function $f(x)$ can be written as the following power series: $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2^n n!}$, find the series representation (using the Σ notation) for the following functions:

(a) (1 pts) $g(x) = 4x^2 f(x^3)$

$g(x) = \sum \square \, x^{\square}$

(b) (2 pts) $h(x) = \frac{d}{dx}\left[f(x^3)\right]$

$h(x) = \sum \square \, x^{\square}$

(c) (2 pts) $w(x) = \int x\, f(x)\, dx$ [You do NOT need to compute the constant C].

$w(x) = \sum \square \, x^{\square}$

8. (10 pts) Application of series: A patient without any previous treatment is injected every 24 hrs with a drug. Immediately before each injection the concentration of the remaining drug in the patient is reduced by 80% from the previous day. Each injection increases the patient drug concentration by 2 mg/L. Denote by C_n the concentration immediately AFTER each injection. Answer the following questions:

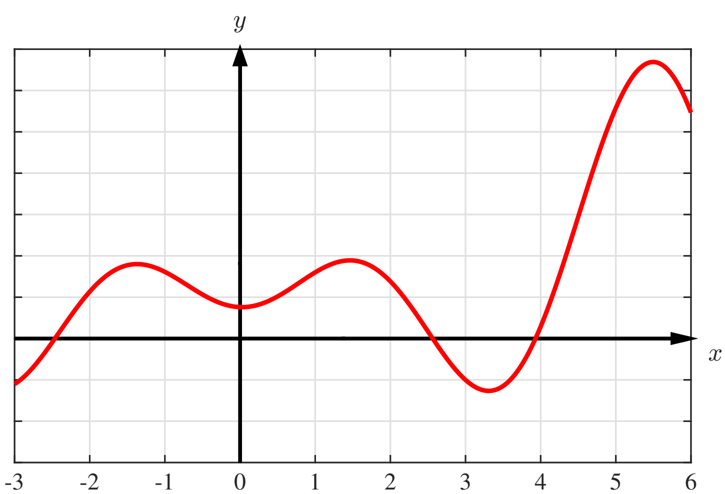
(a) Write explicit expressions for C_1, C_2 and C_3 (the concentrations after the first, second and third injections):

(b) Write an explicit formula, using Σ notation, for the concentration C_n (just after the n-th injection) as a function of n.

(c) If the patient remains indefinitely with this treatment, what would be the limiting value of the concentration?

9. (5 pts) Suppose that you are given the graph of the function $f(x)$ depicted on the right. Let us denote $T_n(x)$ the Taylor series approximation of order n [$n = 0$ denotes a constant, $n = 1$ denotes a LINEAR approximation, $n = 2$ denotes a CUADRATIC approximation, etc...]. Sketch the graphs of the following Taylor series approximations:

(a) T_0 at $\boldsymbol{x = 0}$ (use a **thin solid** line).
(b) T_2 at $\boldsymbol{x = 0}$ (use a **dashed** line).
(c) T_0 at $\boldsymbol{x = 3}$ (use a **thin solid** line).
(d) T_1 at $\boldsymbol{x = 3}$ (use a **dashed** line).
(e) T_2 at $\boldsymbol{x = 5.5}$ (use a **dashed** line).



10. EXTRA CREDIT (5 pts):
L'Hopital rule: If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$, one can compute the limit $\lim_{x\to a} \dfrac{f(x)}{g(x)}$ by computing $\lim_{x\to a} \dfrac{f'(x)}{g'(x)}$.
By Taylor expanding (about $x = a$) f and g, prove L'Hopital rule and give the conditions when it can used.