Midterm #3 (v1) — Math 151 — Calculus II — Fall 2018

I, ____, student of section ________, pledge that this material is completely my own work, and that I did not take, borrow, or copy any portions from any other person(s). I understand if I violate this honesty pledge, I am subject to disciplinary actions pursuant to the appropriate sections of the San Diego State University Policies.

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Signature

(0) Write your first and last name above using CAPITAL LETTERS.
(1) If you use pencil please **use pressure!!!**
If you write softly with pencil the scan will be unreadable and your test will NOT be graded!
(2) Do NOT alter the QR-code above! If you do so, your paper will not be graded and you will get a ZERO.
(3) Do NOT open this test booklet until told to do so.
(4) Do ALL your work on this test booklet.
(5) If you need extra space please use the last page.
(6) NO CALCULATORS, NO CHEAT-SHEETS or any other aids allowed.
(7) You may write in either pen or pencil, but answers deemed illegible will be ignored.
(8) Please enter your answers in the BOXES provided
(9) Please check that all **8 pages** (including this cover sheet) are intact.
(10) The value for each question is given in the table below.
(11) In all the questions you should indicate how you arrived at your answer.
(12) To get full credit you need to simplify your answers (cf. $\sin(0) = 0, e^0 = 1, \sqrt{4} = 2, 2/4 = 1/2$, etc...).

1	2	3	4	5	6	7	8	9	xtr	Total
/8	/8	/20	/8	/6	/8	/8	/5	/8	/4	/79

Do NOT write ANYTHING above this line!

1. (8 pts) Using the **ratio test**, determine the **radius** AND the **interval** of convergence of the following infinite series. **Do NOT study convergence at the end points.** Explain what you are doing and show all your work!

$$S_1 = \sum_{n=0}^{\infty} (-1)^{n+1} \frac{(n+1)^2}{5^n} (x+2)^n$$

Radius of conv.:	Interval of conv.: $< x <$

2. (8 pts) Determine whether the following infinite series **converges absolutely**, **converges conditionally** or **diverges**. For convergence use the **alternating series test** and for absolute convergence use the **limit comparison test**. You must show all your work. No detailed explanations → no points!

$$S_2 = \sum_{n=1}^{\infty} (-1)^n \frac{n+1}{2n^2}$$

Circle one:	diverges	converges absolutely	converges conditionally

Do NOT write ANYTHING above this line!

3. (20 pts) Determine whether the following infinite series converge or diverge USING THE INDICATED TEST. Make sure to check that ALL conditions for each test are satisfied. **No detailed explanations → no points!!!**

a) (5 pts) $S_3 = \sum_{n=1}^{\infty} \frac{\cos(3n) + 2}{n^2}$ (Direct comparison [sandwich] test)

b) (5 pts) $S_4 = \sum_{n=1}^{\infty} \frac{5n^2}{4 + 3n^2}$ (Divergence test)

c) (5 pts) $S_5 = \sum_{n=2}^{\infty} \frac{4}{\sqrt{3n - 1}}$ (Integral test)

d) (5 pts) $\boldsymbol{S_6 = \sum_{n=2}^{\infty} \frac{3\sqrt{n+6}}{n^3-4}}$ (Limit comparison test)

4. (8 pts) What can you say about the convergence of $\sum_{n=1}^{\infty} a_n$ in each of the following cases. Circle ONE option.

(i) $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = \pi^2$	$\sum_{n=1}^{\infty} a_n$ is: Convergent Divergent Inconclusive
(ii) $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = 1$	$\sum_{n=1}^{\infty} a_n$ is: Convergent Divergent Inconclusive
(iii) $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = e^{-2}$	$\sum_{n=1}^{\infty} a_n$ is: Convergent Divergent Inconclusive
(iv) $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$ and $a_n > 0$ and $b_n > 0$ and $\sum_{n=1}^{\infty} b_n$ converges.	$\sum_{n=1}^{\infty} a_n$ is: Convergent Divergent Inconclusive
(v) $\lim_{n\to\infty} a_n = 0$.	$\sum_{n=1}^{\infty} a_n$ is: Convergent Divergent Inconclusive
(vi) $0 \leq a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ converges.	$\sum_{n=1}^{\infty} a_n$ is: Convergent Divergent Inconclusive
(vii) $0 \leq b_n \leq a_n$ and $\sum_{n=1}^{\infty} b_n$ converges.	$\sum_{n=1}^{\infty} a_n$ is: Convergent Divergent Inconclusive
(viii) $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.	$\sum_{n=1}^{\infty} a_n$ is: Convergent Divergent Inconclusive

Do NOT write ANYTHING above this line!

5. **(6 pts)** Using the fact that function $f(x)$ can be written by the following series: find the series representation (using the Σ notation) for the following functions: $\boldsymbol{f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n}}{4^n n!}}$,

(a) **(2 pts)** $\boldsymbol{g(x) = 3\, f(x^2)}$

$g(x) = \sum^{\infty}$ $n =$ [ ]

(b) **(2 pts)** $\boldsymbol{h(x) = \frac{d}{dx}\left[4\, x\, f(x^3)\right]}$

$h(x) = \sum^{\infty}$ $n =$ [ ]

(c) **(2 pts)** $\boldsymbol{w(x) = \int x\, f(x^4)\, dx}$ [The constant C is already written for you].

$w(x) = \sum^{\infty}$ $n =$ [ ] $+\, \boldsymbol{C}$

6. **(8 pts)** Compute the Taylor polynomial of order 4 (i.e. fourth degree polynomial) for $\boldsymbol{g(x) = B\ \cos(a\, x)}$ about $\boldsymbol{x = 0}$.

$g(x) \approx$

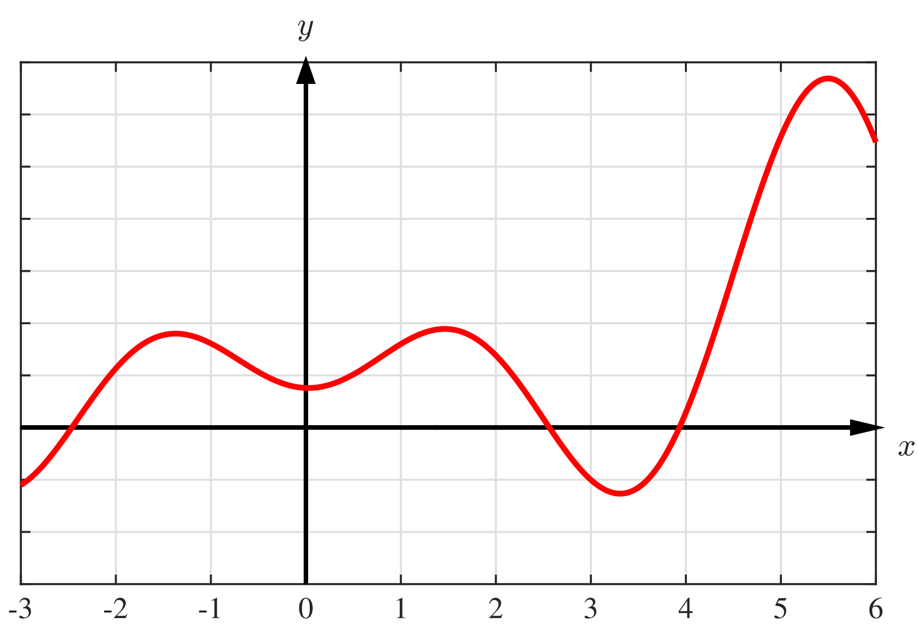
Do NOT write ANYTHING above this line!

7. (8 pts) Compute the Taylor polynomial of order 2 (i.e. second degree polynomial) for $\boldsymbol{f(x) = \sqrt{x-2}}$ about $\boldsymbol{x = 4}$.

$f(x) \approx$

8. (5 pts) Suppose that you are given the graph of the function $f(x)$ depicted on the right. Let us denote $T_n(x)$ the Taylor polynomial approximation of order n [$n = 0$ denotes a constant, $n = 1$ denotes a LINEAR approximation, $n = 2$ denotes a QUADRATIC approximation, etc...]. Sketch the graphs of the following Taylor approximations:

(a) T_0 at $\boldsymbol{x = 0}$ (use a **thin solid** line).
(b) T_2 at $\boldsymbol{x = 0}$ (use a **dashed** line).
(c) T_0 at $\boldsymbol{x = 3}$ (use a **thin solid** line).
(d) T_1 at $\boldsymbol{x = 3}$ (use a **dashed** line).
(e) T_2 at $\boldsymbol{x = 5.5}$ (use a **dashed** line).



9. (8 pts) A patient starts taking **800 mg** of a drug at the same time every day. After each day, just before the next tablet is taken, some of the drug has been metabolized so that **20%** of the drug remains in the body. The patient starts taking the drug for the first time on day $n = 1$. Answer the following questions:

(a) What quantity, q_n, of the drug is in the body **JUST AFTER** taking the (i) first tablet: q_1, (ii) just after taking the second tablet: q_2, (iii) just after taking the third tablet: q_3, and (iii) just after taking the fourth tablet: q_4?
Write each results as a NUMBER!

- Just after taking 1st tablet : $\boldsymbol{q_1} =$ ______ mg
- Just after taking 2nd tablet : $\boldsymbol{q_2} =$ ______ $=$ ______ mg
- Just after taking 3rd tablet : $\boldsymbol{q_3} =$ ______ $=$ ______ mg
- Just after taking 4th tablet : $\boldsymbol{q_4} =$ ______ $=$ ______ mg

(b) After identifying the pattern that you obtain in the previous question, write a formula USING SUMMATION NOTATION for the quantity q_n of the drug that is in the body **just after** taking the n-th tablet?

(c) What quantity of the drug remains in the body **just after** taking the next tablet in the long run?
Simlify your result an write it as an **INTEGER NUMBER**!

10. EXTRA CREDIT (4 pts):
L'Hopital rule: If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$, one can compute the limit $\lim_{x\to a} \dfrac{f(x)}{g(x)}$ by computing $\lim_{x\to a} \dfrac{f'(x)}{g'(x)}$.
By Taylor expanding (about $x = a$) f and g, prove L'Hopital rule and give the conditions when it can used.

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