Do NOT write ANYTHING above this line!

Midterm #3 (v1) — Math 151 — Calculus II — Fall 2019

I, ____, student of section ________, pledge that this material is completely my own work, and that I did not take, borrow, or copy any portions from any other person(s). I understand if I violate this honesty pledge, I am subject to disciplinary actions pursuant to the appropriate sections of the San Diego State University Policies.

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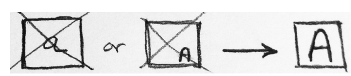
Signature

(0) Write your first and last name above using **LARGE CAPITAL LETTERS**: 

(1) If you use pencil please **use pressure!!!**
If you write softly with pencil the scan will be unreadable and your test will NOT be graded!
(2) Do NOT alter the QR-code above! If you do so, your paper will not be graded and you will get a ZERO.
(3) Do NOT open this test booklet until told to do so.
(4) Do ALL your work on this test booklet.
(5) NO CALCULATORS, NO CHEAT-SHEETS or any other aids allowed.
(6) You may write in either pen or pencil, but answers deemed illegible will be ignored. (see point#1 above)
(7) Please enter your answers in the BOXES provided
(8) Please check that all **8 pages** (including this cover sheet) are intact.
(9) The value for each question is given in the table below.
(10) In all the questions you should indicate how you arrived at your answer.
(11) To get full credit you need to simplify your answers (cf. $\sin(0) = 0, e^0 = 1, \sqrt{4} = 2, 2/4 = 1/2$, etc...).

1	2	3	4	5	6	7	8	9	10	Total
/8	/4	/20	/6	/8	/4	/6	/6	/6	/6	/74

1. (8 pts) Solve the following differential equation satisfying the given initial condition. $\boldsymbol{A}$ and $\boldsymbol{B}$ are **fixed** constants.
(i) Give first GENERAL solution and the **(ii)** the PARTICULAR solution satisfying the initial condition.

$\dfrac{dy}{dx} - 2B\,x\,y - 2\,e^{Bx^2} = 0$ with $y(0) = A$.

(i) Gral sol: $y(x) =$

(ii) Part sol: $y(x) =$

2. (4 pts) Sequences

(a) [**2 pts**] Find a formula for the general term a_n (**starting at** $\boldsymbol{n = 1}$) for the sequence: $\left\{3, -\frac{6}{2}, \frac{9}{6}, -\frac{12}{24}, \frac{15}{120}, \ldots\right\}$

(b) [**2 pts**] Determine whether the following **sequence** converges or diverges. If it converges, find the limit.

$\{a_n\}_{n=1}^{\infty} = \left\{\dfrac{n^2(n+7)}{2n^3}\right\}_{n=1}^{\infty}$

3. (20 pts) Determine whether the following infinite series converge or diverge USING THE INDICATED TEST. Make sure to **STATE** and **CHECK** that **ALL** conditions for each test are satisfied.
No detailed explanations → no points!!!

a) (5 pts) $\boldsymbol{S_1 = \sum_{n=1}^{\infty} \frac{e^{-n}+3}{5+n^2}}$ (Direct comparison test)

b) (5 pts) $\boldsymbol{S_2 = \sum_{n=1}^{\infty} \frac{2n^2\sqrt{3n}}{4+5n^{5/2}}}$ (Divergence test)

c) (5 pts) $\boldsymbol{S_5 = \sum_{n=2}^{\infty} \frac{4n}{\sqrt{3n^2-1}}}$ (Integral test)

d) (5 pts) $S_4 = \displaystyle\sum_{n=2}^{\infty} \frac{3n + \cos(n)}{2n^3 - 2}$ (Limit comparison test)

4. (6 pts) Using the **ratio test**, determine the **radius** AND the **interval** of convergence of the following infinite series. **Do NOT study convergence at the end points.** Explain what you are doing and show all your work!

$$\sum_{n=3}^{\infty} (-1)^n \frac{n^2}{5^n} (x-2)^n$$

Radius of conv.:	Interval of conv.: $< x <$

5. (8 pts) Determine whether the following infinite series **converges absolutely**, **converges conditionally** or **diverges**. For absolute convergence use the **limit comparison test** and for convergence use the **alternating series test**. You must show all your work. **No detailed explanations → no points!**

$$S_5 = \sum_{n=0}^{\infty} (-1)^n \frac{3n}{4n^2+2}$$

(a) Absolute convegence: (use limit comparison test)

(b) Convegence: (use alternating series test)

(c) Conclusion:

Circle one:	diverges	converges absolutely	converges conditionally

6. (4 pts) What can you say about the convergence of $\sum_{n=1}^{\infty} a_n$ in each of the following cases. Circle ONE option.

(i) $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = \frac{10}{\pi^2}$	$\sum_{n=1}^{\infty} a_n$ is: Convergent Divergent Inconclusive
(ii) $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = e^0$	$\sum_{n=1}^{\infty} a_n$ is: Convergent Divergent Inconclusive
(iii) $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = \cos(1)$	$\sum_{n=1}^{\infty} a_n$ is: Convergent Divergent Inconclusive
(iv) $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$ and $a_n > 0$ and $b_n > 0$ and $\sum_{n=1}^{\infty} b_n$ converges.	$\sum_{n=1}^{\infty} a_n$ is: Convergent Divergent Inconclusive
(v) $\lim_{n\to\infty} a_n = 0$.	$\sum_{n=1}^{\infty} a_n$ is: Convergent Divergent Inconclusive
(vi) $0 \le a_n \le b_n$ and $\sum_{n=1}^{\infty} b_n$ converges.	$\sum_{n=1}^{\infty} a_n$ is: Convergent Divergent Inconclusive
(vii) $0 \le b_n \le a_n$ and $\sum_{n=1}^{\infty} b_n$ converges.	$\sum_{n=1}^{\infty} a_n$ is: Convergent Divergent Inconclusive
(viii) $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.	$\sum_{n=1}^{\infty} a_n$ is: Convergent Divergent Inconclusive

7. (6 pts) Using the fact that function $f(x)$ can be written by the following series: find the series representation (using the Σ notation) for the following functions:

$$\boldsymbol{f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n}}{4^n}},$$

(a) (2 pts) $\boldsymbol{g(x) = 3\, f(x^2)}$

$$g(x) = \sum_{\boxed{n =\quad}}^{\infty} \boxed{\qquad\qquad}$$

(b) (2 pts) $\boldsymbol{h(x) = \frac{d}{dx}\left[4\, x\, f(x^3)\right]}$

$$h(x) = \sum_{\boxed{n =\quad}}^{\infty} \boxed{\qquad\qquad}$$

(c) (2 pts) $\boldsymbol{w(x) = \int x\, f(x^4)\, dx}$ [The constant C is already written for you].

$$w(x) = \sum_{\boxed{n =\quad}}^{\infty} \boxed{\qquad\qquad} + \boldsymbol{C}$$

8. (6 pts) Compute the Taylor polynomial of order 3 (i.e. third degree polynomial) for $\boldsymbol{f(x) = \sqrt{x+2}}$ about $\boldsymbol{x = 0}$.

$f(x) \approx$

9. (6 pts) Compute the Taylor polynomial of order 3 (i.e. third degree polynomial) for $\boldsymbol{g(x) = B\, e^{2x}}$ about $\boldsymbol{x = A}$.

$g(x) \approx$

10. (6 pts) A tank initially contains M_0 mg of air. The air is removed by the strokes of a pump. During each pump stroke: (i) 50% of the air is removed and then (ii) 5 mg of air leaks back into the tank.
Note: first the air is removed and only then some air leaks back in.

(a) Compute, **as function of $\boldsymbol{M_0}$**, the amount of air after one pump cycle (M_1), after two pump cycles (M_2), three pump cycles (M_3), and four pump cycles (M_4):

$M_1 =$

$M_2 =$

$M_3 =$

$M_4 =$

(b) Using the pattern above, write M_i as a function of M_0 for general i using the sum (Σ) notation.

(c) How much air will remain in the tank after pumping indefinitely?
Extra credit (+1pt): How does this depend on the initial amount of air (M_0)? Explain if your result makes sense.