Midterm #3 (v1) — Math 151 — Calculus II — Spring 2018

Professor/TA: ______________________ **Sec:** ________ **RedID:** ______________

NAME (printed): ______________________ ______________________

(Last Name) **(First Name)**

I, ______________________, pledge that this material is completely my own work, and that I did not take, borrow, or copy any portions from any other person(s). I understand if I violate this honesty pledge, I am subject to disciplinary actions pursuant to the appropriate sections of the San Diego State University Policies.

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Signature

(1) Do NOT open this test booklet until told to do so.
(2) Do ALL your work on this test booklet.
(3) If you need extra space please ask instructor for extra paper.
(4) NO CALCULATORS, NO CHEAT-SHEETS or any other aids allowed.
(5) You may write in either pen or pencil, but answers deemed illegible will be ignored.
(6) Please enter your answers in the BOXES provided
(7) Please check that all **6 pages** (including this cover sheet) are intact.
(8) The value for each question is given in the table below.
(9) In all the questions you should indicate how you arrived at your answer.
(10) To get full credit you need to simplify your answers (cf. $\sin(0) = 0, e^0 = 1, \sqrt{4} = 2, 2/4 = 1/2$, etc...).

1	2	3	4	5	6	7	8	9	10	Total
/10	/10	/10	/10	/10	/ 6	/10	/ 5	/ 5	/ 5	/81

1. (10 pts) Solve the following differential equation satisfying the given initial conditions. $\boldsymbol{A}$ and $\boldsymbol{B}$ are fixed constants. **(i)** Give first GENERAL solution and the **(ii)** the PARTICULAR solution satisfying the initial condition.
Hint: $\boldsymbol{e^{\alpha \ln x} = e^{\ln x^{\alpha}} = x^{\alpha}}$.

$\boldsymbol{x\, y' + A\, y = B\, x^2}$ with $\boldsymbol{y(1) = 0}$.

(i) Gral sol: $y(x) =$

(ii) Part sol: $y(x) =$

2. (10 pts) Using the **ratio test**, determine the **radius of convergence** AND the **interval of convergence** of the following infinite series. **Do NOT study convergence at the end points.** Explain what you are doing and show all your work!

$$\sum_{n=3}^{\infty} (-1)^n \, \frac{n^3}{5^n} \, (x-2)^n$$

Radius of convergence:

Interval of convergence:

3. (10 pts) Determine whether the following infinite series converge or diverge USING THE INDICATED TEST. Make sure to check that ALL conditions for each test are satisfied. **No detailed explanations → no points!!!**

a) (5 pts) $S_1 = \sum_{n=1}^{\infty} \frac{\cos^2(3n)}{2+n^2}$ (Direct comparison test)

b) (5 pts) $S_2 = \sum_{n=1}^{\infty} \frac{4\sqrt{n}}{\sqrt{2+5n}}$ (Divergence test)

4. (10 pts) Determine whether the following infinite series converge or diverge USING THE INDICATED TEST. Make sure to check that ALL conditions for each test are satisfied. **No detailed explanations → no points!!!**

a) (5 pts) $S_3 = \sum_{n=2}^{\infty} \frac{5n}{\sqrt{n^2-2}}$ (Integral test)

b) (5 pts) $S_4 = \sum_{n=2}^{\infty} \frac{3n+5}{2n^3-2}$ (Limit comparison test)

5. (10 pts) Determine whether the following infinite series **converges absolutely**, **converges conditionally** or **diverges**. For convergence use the **alternating series test** and for absolute convergence use the **limit comparison test**. You must show all your work. No detailed explanations → no points!

$$S_5 = \sum_{n=1}^{\infty} (-1)^n \frac{n^3}{n^4 + 3}$$

Circle one:	diverges	converges absolutely	converges conditionally

6. (6 pts) Using the fact that the function $f(x)$ can be written as the following power series: $f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{2^n}$, find the series representation (using the Σ notation) for the following functions:

(a) (2 pts) $g(x) = 3\,x^3\, f(x^2)$

$$g(x) = \sum_{n=\quad}^{\infty} \boxed{}$$

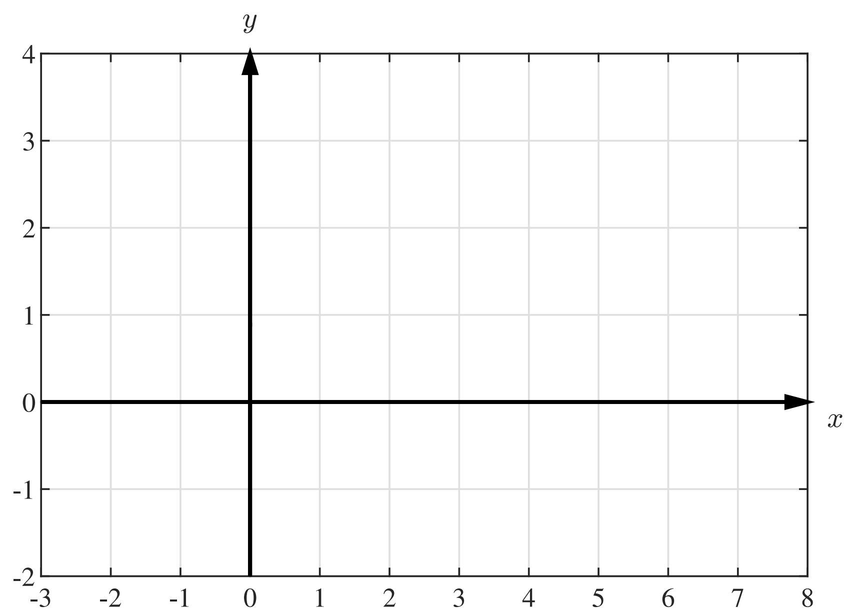
(b) (2 pts) $h(x) = \dfrac{d}{dx}\,[f(x)]$

$$h(x) = \sum_{n=\quad}^{\infty} \boxed{}$$

(c) (2 pts) $w(x) = \displaystyle\int f(x)\,dx$ [You do NOT need to compute the constant C].

$$w(x) = \sum_{n=\quad}^{\infty} \boxed{}$$

7. (10 pts) (a) Compute the Taylor polynomial of order 2 (i.e. second degree polynomial) for $\boldsymbol{f(x) = \ln(x+2)}$ about $\boldsymbol{x = -1}$. [You do not need to do any convergence]. **(b)** Qualitatively sketch (i) the function (bold curve), its (ii) linear approximation $L(x)$ (thin curve), and (iii) its quadratic approximation $Q(x)$ (dashed curve) and label each curve accordingly. [You can use the following values: $\ln(\frac{1}{4}) \approx -1.4$, $\ln(\frac{1}{2}) \approx -0.7$, $\ln(2) \approx 0.7$, $\ln(4) \approx 1.4$, $\ln(6) \approx 1.8$, $\ln(8) \approx 2.1$, $\ln(10) \approx 2.3$].

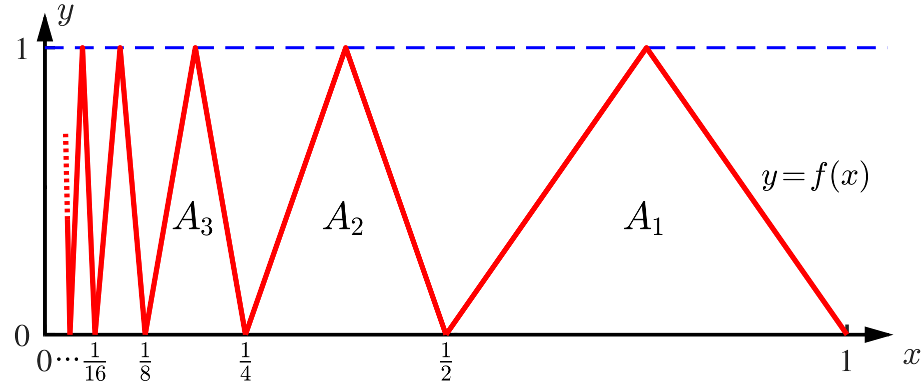


$f(x) \approx$

8. (5 pts) Compute the Taylor polynomial of order 4 (i.e. fourth degree polynomial) for $\boldsymbol{g(x) = B\, e^{2x}}$ about $\boldsymbol{x = \alpha}$.

$g(x) \approx$

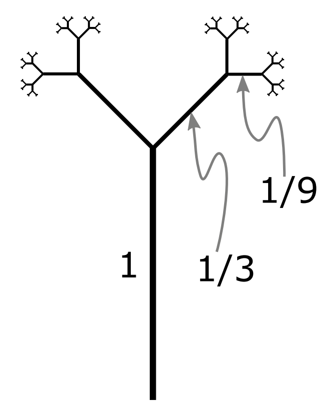
9. **(5 pts)** Evaluate the integral $I_1 = \int_0^1 f(x)\,dx$, where f is the function whose graph is shown below.
Hints: (i) The area of a triangle is $\frac{1}{2}$(base×height). (ii) Compute the areas A_1, A_2, etc..., find a pattern and sum them up!



$I_1 =$

10. **(5 pts)** Many plants and animals have developed roots and vascular systems that optimize the intake/exchange of environmental resources. This has lead to many of these system to take fractal shapes. Assume we have a branching system where each mother branch splits into **TWO** daughter branches and so on as depicted in the figure.

Assume in our case that the main mother branch has a length $L_0 = 1$ and that all daughter branches have a length L_{i+1} that is 1/3 of their mother branch length L_i [i.e. $L_{i+1} = L_i/3$]. Compute the TOTAL LENGTH of this branch system (including ALL branches) after an infinite number of splits. [Note that there is only one mother branch!]



$L =$