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Midterm #3 (v2) — Math 151 — Calculus II — Spring 2019

I, _____, student of section _____, pledge that this material is completely my own work, and that I did not take, borrow, or copy any portions from any other person(s). I understand if I violate this honesty pledge, I am subject to disciplinary actions pursuant to the appropriate sections of the San Diego State University Policies.

_____

Signature

(0) Write your first and last name above using **LARGE CAPITAL LETTERS**.
(1) If you use pencil please **use pressure!!!**
If you write softly with pencil the scan will be unreadable and your test will NOT be graded!
(2) Do NOT alter the QR-code above! If you do so, your paper will not be graded and you will get a ZERO.
(3) Do NOT open this test booklet until told to do so.
(4) Do ALL your work on this test booklet.
(5) NO CALCULATORS, NO CHEAT-SHEETS or any other aids allowed.
(6) You may write in either pen or pencil, but answers deemed illegible will be ignored. (see point#1 above)
(7) Please enter your answers in the BOXES provided
(8) Please check that all **8 pages** (including this cover sheet) are intact.
(9) The value for each question is given in the table below.
(10) In all the questions you should indicate how you arrived at your answer.
(11) To get full credit you need to simplify your answers (cf. $\sin(0) = 0, e^0 = 1, \sqrt{4} = 2, 2/4 = 1/2$, etc...).

1	2	3	4	5	6	7	8	9	xtr	Total
/5	/8	/20	/8	/8	/8	/8	/8	/5	/3	/81

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1. (5 pts) Sequences

(a) [1 pts] What is the difference/relation between a series and a sequence?

(b) [2 pts] Find a formula for the general term a_n (**starting at $n = 1$**) for the sequence: $\left\{\frac{5}{1}, -\frac{10}{4}, \frac{20}{7}, -\frac{40}{10}, \frac{80}{13}, \ldots\right\}$

(c) [2 pts] Determine whether the following **sequence** converges or diverges. If it converges, find the limit.
$\{a_n\}_{n=1}^{\infty} = \left\{4\, e^{-3n}\, n^2\right\}_{n=1}^{\infty}$

2. (8 pts) Solve the following differential equation satisfying the given initial conditions. A and B are fixed constants. **(i)** Give first GENERAL solution and the **(ii)** the PARTICULAR solution satisfying the initial condition.
Hint: $e^{\alpha \ln x} = e^{\ln x^{\alpha}} = x^{\alpha}$.

$x\, y' + B\, y = A\, x^4$ with $y(1) = 0$.

(i) Gral sol: $y(x) =$

(ii) Part sol: $y(x) =$

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3. (20 pts) Determine whether the following infinite series converge or diverge USING THE INDICATED TEST. Make sure to **STATE** and **CHECK** that **ALL** conditions for each test are satisfied.
No detailed explanations → no points!!!

a) (5 pts) $S_1 = \sum_{n=1}^{\infty} \frac{\sin^2(2n)+1}{3+n^2}$ (Direct comparison test)

b) (5 pts) $S_2 = \sum_{n=1}^{\infty} \frac{3\sqrt{2+n^2}}{\sqrt{4+2n+7n^2}}$ (Divergence test)

c) (5 pts) $S_3 = \sum_{n=3}^{\infty} \frac{7n}{\sqrt{n^2-1}}$ (Integral test)

d) (5 pts) $\boldsymbol{S_4 = \sum_{n=1}^{\infty} \frac{4^n + 3^n}{n^2\ 4^n}}$ (Limit comparison test)

4. (8 pts) Using the **ratio test**, determine the **radius** AND the **interval** of convergence of the following infinite series. **Do NOT study convergence at the end points.** Explain what you are doing and show all your work!

$$\boldsymbol{S_5 = \sum_{n=1}^{\infty} (-1)^n \frac{n^2}{3^n} (5x + 2)^n}$$

Radius of conv.:	Interval of conv.: $< x <$

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5. (8 pts) Determine whether the following infinite series **converges absolutely**, **converges conditionally** or **diverges**. For absolute convergence use the **limit comparison test** and for convergence use the **alternating series test**. You must show all your work. **No detailed explanations → no points!**

$$S_6 = \sum_{n=1}^{\infty} (-1)^n \frac{n^2}{5n^3 + 2}$$

(a) Absolute convegence: (use limit comparison test)

(b) Convegence: (use alternating series test)

(c) Conclusion:

Circle one:	diverges	converges absolutely	converges conditionally

6. (8 pts) Compute the Taylor polynomial of order 2 (i.e., second degree polynomial) for $\boldsymbol{f(x) = \ln(x+3)}$ about $\boldsymbol{x = -2}$.

$f(x) \approx$

7. (8 pts) Compute the Taylor polynomial of order 3 (i.e. third degree polynomial) for $\boldsymbol{g(x) = Bx + C\, e^{Ax}}$ about $\boldsymbol{x = 0}$.

$g(x) \approx$

8. (8 pts) Assume you have a savings bank account that initially contains M_0 **dollars**. After each year, you deposit a constant amount of **200 dollars** into the account. The savings account yields a **4%** gains every year.

(a) Compute, **as function of M_0**, the dollar amount that you will have after **one year** (M_1), **two years** (M_2), and **three years** (M_3):

$M_1 =$

$M_2 =$

$M_3 =$

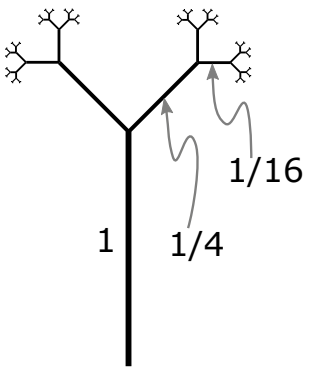
(b) Using the pattern above, write M_n (the amount that you have after n years) as a function of M_0 for general n using the sum (Σ) notation. Be careful to indicate which terms are INSIDE the sum and which ones are OUTSIDE of the sum!

(c) Rewrite M_n using the following partial geometric sum: $\sum_{i=0}^{n-1} b\, r^i = b\, \frac{r^n - 1}{r - 1}$.

(d) How much money will you have after **25 years** if you initially invested $M_0 = 2000$ dollars? No need of calculator, just leave your answer as an explicit formula (i.e., your final answer should not have any variables and only have numbers).

(e) How much money will you have if you keep investing forever? [Explain in detail!]

9. (5 pts) Many plants and animals have developed roots and vascular systems that optimize the intake/exchange of environmental resources. This has lead to many of these system to take fractal shapes. Assume we have a branching system where each mother branch splits into **TWO** daughter branches and so on as depicted in the figure.



Assume in our case that the main mother branch has a length $\ell_0 = 1$ and that all daughter branches have a length ℓ_{i+1} that is 1/4 of their mother branch length ℓ_i [i.e. $\ell_{i+1} = \ell_i/4$]. Compute the TOTAL LENGTH L of this branch system (including ALL branches) after an infinite number of splits. [Note that there is only one mother branch!]

$L =$

10. Extra credit:
(3 pts) Using the fact that, for $|r| < 1$, $\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$, prove that $\sum_{n=A}^{\infty} r^{n+C} = \frac{\text{FIRST}}{1-r}$ where A and C are any integers and FIRST is the first term of the original series.