Mini-Test #2 (v1) — Math 151 — Calculus II — Spring 2021

You have **ONE hour** (+ 10 mins) to submit your answers for this mini-test. Please strictly adhere to the following instructions:

(1) Write a **FULL** solution for **each** problem on a *separate* piece of paper. You need to use the methods that were taught in class and you need to show how you arrived to your answer. Failing to provide all the details on how you arrived to your answer will be deemed as suspicious and you risk being subject to disciplinary actions (in addition of getting an F in the whole test).

(2) Start each solution, on a *separate* piece of paper, by writing the question number. Write clearly/neatly and **BOX your final answers**. If you do not box your final answer, your answer will NOT be graded or you will get points deducted!

(3) When you are ready to submit, and **no later than 60 minutes after the start of the test**, collect all your answers into a **single PDF** and upload by **matching** the different pages of your PDF to the questions in the test. Each failure to match the correct problem will incur a point deduction.

(4) If the problem includes a figure: please reproduce the figure carefully so that you can use it to solve the question.

(5) Make sure to always upload pics/images that are not blurry and that are oriented correctly (an upside down or blurry pic earns NO points. Seriously!).

(6) Only use techniques that were taught in class and make sure that all of your answers are accompanied by their respective explanations. **No full work shown = no points.**

Here is an honor pledge that you need to sign and date. Failure to sign will automatically result in a zero for this test:

Question#1. HONOR PLEDGE:

(A) The material that I am uploading is **completely my own work**, and that I did not take, borrow, or copy any portions from ANY other sources. This includes, but it is not limited to, NOT using any of the following resources: calculator, internet [Chegg, Slader, WolframAlpha, IntegralCalculator, WhatsApp, Instagram, etc], cellpone, computer, roommate, friend, tutor, etc...

(B) I will NEVER post/share/send/upload/download (DURING or AFTER the test) ANY portions of this test to/from the internet or any other type of platform.

(C) I will **stop solving the test after 60mins** and will use the last 10mins for uploading. **It is MY responsibility to upload before the time runs out.** If I run out of time I will NOT contact the calc team for help.

I understand if I violate this honesty pledge, I will earn an F for the whole semester and I will be subject to disciplinary actions pursuant to the appropriate sections of the San Diego State University Policies.

WRITE BELOW:

"**I have read and understood all of the above points.**" and then write your name, sign, write your RedID, your section number, professor, and date:

I ____

First/last name	Signature	RedID	Sec.#	Professor	date

2. **(4 pts) Arclength:** Write BOTH an $\boldsymbol{x}$ AND a $\boldsymbol{y}$ integral for the length of the curve

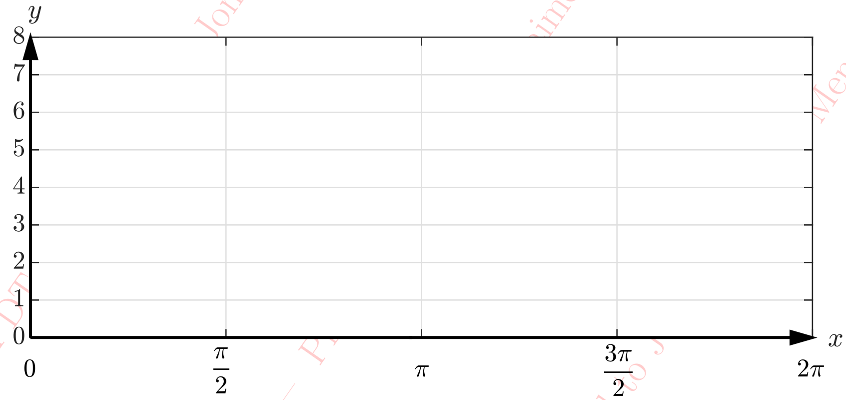
$$y = f(x) = 3 + 2\cos x \quad \text{for} \quad 0 \le x \le 2\pi.$$

Graph the function. You do NOT need to compute the integral! Hint#1: you may need one of these derivatives:

- $[\arccos x]' = [\cos^{-1} x]' = -\dfrac{1}{\sqrt{1-x^2}}$
- $[\arcsin x]' = [\sin^{-1} x]' = \dfrac{1}{\sqrt{1-x^2}}$

Hint#2: for the y integral you might want to use the symmetry of the function.

[SDSU M151 S21 MiniTest2 V1 Q2 19/Mar/2021 5:00-6:10pm PDT — Do NOT share/distribute/post/upload]
[Remember to **stop** solving and **submit** when there are **10 mins** (or more) left on the clock!!! No late submissions!]



Over x: $L_x = \int_{\square}^{\square} \boxed{} \, dx$

Over y: $L_y = \int_{\square}^{\square} \boxed{} \, dy$

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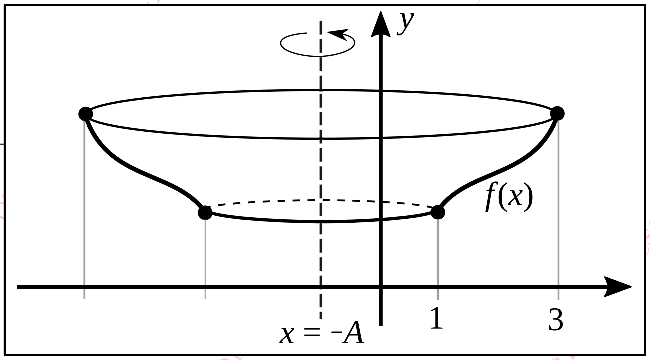
3. **(4 pts) Areas for surfaces of revolution:**
[SDSU M151 S21 MiniTest2 V1 Q3 19/Mar/2021 5:00-6:10pm PDT — Do NOT share/distribute/post/upload]
[Remember to **stop** solving and **submit** when there are **10 mins** (or more) left on the clock!!! No late submissions!]

Write BOTH an x and a y integral for the SURFACE AREA obtained, by rotating about the line $\boldsymbol{x = -A}$, the function $f(x)$ as depicted in the plot.
Note that rotation is NOT about the y-axis!



$S_x = \int_{\square}^{\square} \square \, dx$

$S_y = \int_{\square}^{\square} \square \, dy$

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4. **(2 pts) Improper integrals:**
[SDSU M151 S21 MiniTest2 V1 Q4 19/Mar/2021 5:00-6:10pm PDT — Do NOT share/distribute/post/upload]
[Remember to **stop** solving and **submit** when there are **10 mins** (or more) left on the clock!!! No late submissions!]

Determine whether or not the following improper integrals converge or diverge.
(i) If divergent: say so and prove/explain.
(ii) If convergent: say so and prove/explain AND, if possible, find the value of the integral.
(iii) Please explain!!! No explanation ⇒ NO POINTS!

$$I_1 = \int_3^5 \frac{2}{(x-3)^2}\,dx.$$

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5. **(3 pts) Improper integrals:**
[SDSU M151 S21 MiniTest2 V1 Q5 19/Mar/2021 5:00-6:10pm PDT — Do NOT share/distribute/post/upload]
[Remember to **stop** solving and **submit** when there are **10 mins** (or more) left on the clock!!! No late submissions!]

Let $\mathbf{0 \le g(x) \le 2x}$ for $\mathbf{x \ge 1}$. Using the Comparison Theorem, **(a)** determine whether the integral

$$\boldsymbol{I_2 = \int_2^{\infty} \frac{3 + g(x)}{x^3}\,dx},$$

converges or diverges. **(b)** If convergent give an **upper bound**. Show all work!

(a) Convergence for I_2 :

(b) Upper bound for I_2 :

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6. **(4 pts) Differential Equations:**
[SDSU M151 S21 MiniTest2 V1 Q6 19/Mar/2021 5:00-6:10pm PDT — Do NOT share/distribute/post/upload]
[Remember to **stop** solving and **submit** when there are **10 mins** (or more) left on the clock!!! No late submissions!]

For the following differential equation: $\boldsymbol{2y' - \frac{1}{y} = 0}$

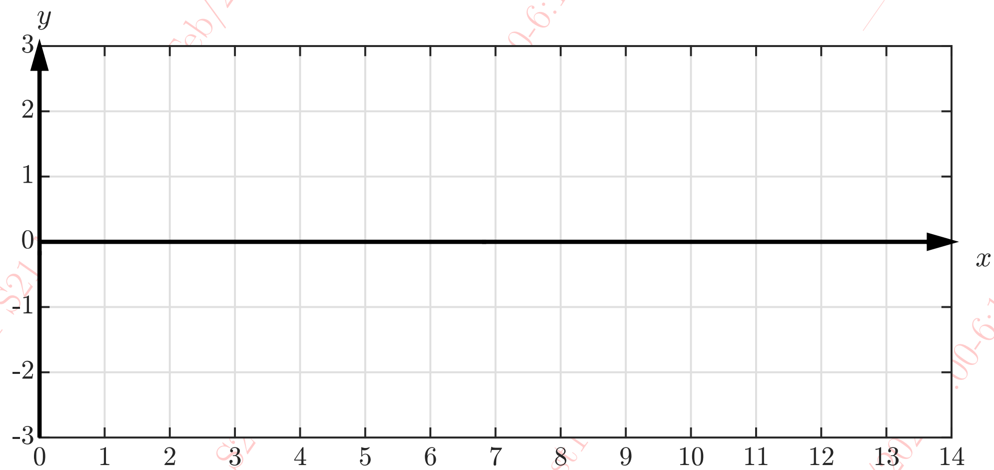
(a) Find the general solution

(a) General sol: $y(x) =$

(b) Find the particular solution satisfying the initial condition: $\boldsymbol{y(6) = -2}$.

(b) Particular sol: $y(x) =$

(c) Sketch a few solutions (encompassing the WHOLE available space in the graph below) using THIN lines and sketch using a THICK line the particular solution you found in (b).



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7. **(3 pts) Applications of Differential Equations:**

[Remember to **stop** solving and **submit** when there are **10 mins** (or more) left on the clock!!! No late submissions!]

Consider a large tank that is initially filled with **10 ℓ** of liquid. Liquid is pumped into the tank at a rate of **20 ℓ/min**. Denote by $\boldsymbol{V}(t)$ the volume [in $\boldsymbol{\ell}$] of liquid in the tank at time $\boldsymbol{t}$. Due to pressure, the talk empties, through a hole on the bottom, at rate $\boldsymbol{r}$ which is *proportional* to the volume of liquid in the tank such that $\boldsymbol{r} = \mathbf{5} \times \boldsymbol{V}(\boldsymbol{t})$ [in $\boldsymbol{\ell}$**/min**].

(a) Write a differential equation for $\boldsymbol{V}(\boldsymbol{t})$.

(b) Find the general solution for this differential equation.

(c) Find the particular solution satisfying the given initial condition.

(extra) Where do ALL solutions tend to when $t \to \infty$? Explain!

EXTRA (3 pts) Applications of Differential Equations: Orthogonal Curves

[SDSU M151 S21 MiniTest2 V1 Q8 19/Mar/2021 5:00-6:10pm PDT — Do NOT share/distribute/post/upload]

[Remember to **stop** solving and **submit** when there are **10 mins** (or more) left on the clock!!! No late submissions!]

For the family of curves $\mathcal{F}$: $\boldsymbol{y = A(x-2)}$ where $\boldsymbol{A}$ is an arbitrary constant:

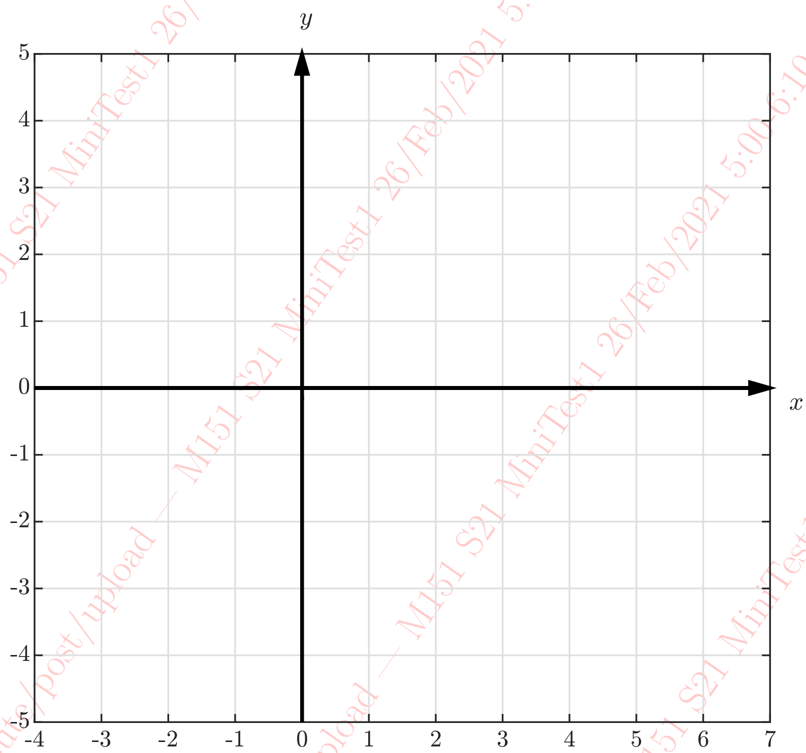
(a) Use DIFFERENTIAL EQUATIONS to find the orthogonal curves to this family.

(b) What geometrical objects do the original family $\mathcal{F}$ and the orthogonal family $\mathcal{F}_\perp$ represent? Be SPECIFIC!

The original family $\mathcal{F}$ is the set of ...

The orthogonal family $\mathcal{F}_\perp$ is the set of ...

(c) Plot a sketch of the two families together (use solid for the original family $\mathcal{F}$ and dashed for the orthogonal family $\mathcal{F}_\perp$).



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