Mini-Test #4 (v1) — Math 151 — Calculus II — Spring 2021

You have **ONE hour** (+ 10 mins) to submit your answers for this mini-test. Please strictly adhere to the following instructions:

(1) Write a **FULL** solution for **each** problem on a *separate* piece of paper. You need to use the methods that were taught in class and you need to show how you arrived to your answer. Failing to provide all the details on how you arrived to your answer will be deemed as suspicious and you risk being subject to disciplinary actions (in addition of getting an F in the whole test).

(2) Start each solution, on a *separate* piece of paper, by writing the question number. Write clearly/neatly and **BOX your final answers**. If you do not box your final answer, your answer will NOT be graded or you will get points deducted!

(3) When you are ready to submit, and **no later than 60 minutes after the start of the test**, collect all your answers into a **single PDF** and upload by **matching** the different pages of your PDF to the questions in the test. Each failure to match the correct problem will incur a point deduction.

(4) If the problem includes a figure: please reproduce the figure carefully so that you can use it to solve the question.

(5) Make sure to always upload pics/images that are not blurry and that are oriented correctly (an upside down or blurry pic earns NO points. Seriously!).

(6) Only use techniques that were taught in class and make sure that all of your answers are accompanied by their respective explanations. **No full work shown = no points.**

(7) Use the method indicated in each problem. Failure to do so earns no points.

Here is an honor pledge that you need to sign and date. Failure to sign will automatically result in a zero for this test:

Question#1. HONOR PLEDGE:

(A) The material that I am uploading is **completely my own work**, and that I did not take, borrow, or copy any portions from ANY other sources. This includes, but it is not limited to, NOT using any of the following resources: calculator, internet [Chegg, Slader, WolframAlpha, IntegralCalculator, WhatsApp, Instagram, etc], cellpone, computer, roommate, friend, tutor, etc...

(B) **I will NEVER post/share/send/upload/download (DURING or AFTER the test) ANY portions of this test to/from the internet or any other type of platform.**

(C) I will **stop solving the test after 60mins** and will use the last 10mins for uploading. **It is MY responsibility to upload before the time runs out.** If I run out of time I will NOT contact the calc team for help.

I understand if I violate this honesty pledge, I will earn an F for the whole semester and I will be subject to disciplinary actions pursuant to the appropriate sections of the San Diego State University Policies.

WRITE BELOW:

"**I have read and understood all of the above points.**" and then write your name, sign, write your RedID, your section number, professor, and date:

I ____

First/last name	Signature	RedID	Sec.#	Professor	date

2. **(2 pts) Power Series:**
[SDSU M151 S21 MiniTest4 V1 Q2 30/Apr/2021 5:00-6:10pm PDT — Do NOT share/distribute/post/upload]
[Remember to **stop** solving and **submit** when there are **10 mins** (or more) left on the clock!!! No late submissions!]

Use the fact that the exponential function $e^x = \sum_{n=0}^{\infty} x^n,$ find the series representation (using the Σ notation) for the following functions:

(a) $g(x) = \dfrac{d}{dx}\left[4\,x\,e^{x^3}\right]$

$g(x) = \sum_{n=0}^{\infty}$ [ ]

(b) $h(x) = \int e^{x^2}\,dx$ [The constant C is already written for you].

$h(x) = \sum_{n=0}^{\infty}$ [ ] $+ C$

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3. **(3 pts) Taylor Series:**
[SDSU M151 S21 MiniTest4 V1 Q3 30/Apr/2021 5:00-6:10pm PDT — Do NOT share/distribute/post/upload]
[Remember to **stop** solving and **submit** when there are **10 mins** (or more) left on the clock!!! No late submissions!]

Below, we depict the graph of the function $y = f(x)$ (thick grey line) together with the graphs of the functions g_i for $i = 1, 2, ..., 7$ (thin solid black lines). As seen in class, we define $T_n(x)$ to be the Taylor polynomial of order n, approximating the function $f(x)$ at the anchoring point $x = a$, as:

$$T_n(x) = \sum_{i=0}^{n} \frac{f^{(i)}}{i!} (x-a)^i.$$

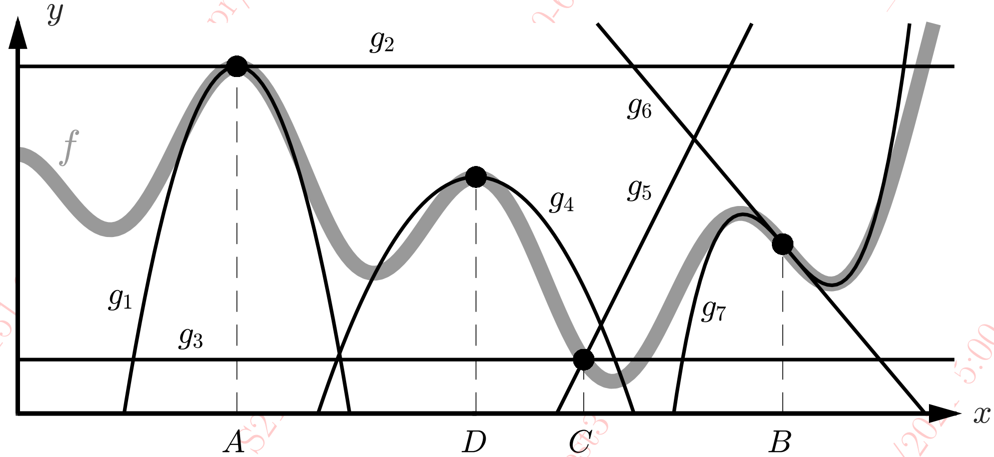
Given the depiction of $y = f(x)$ and $y = g_i(x)$ in the figure below, perform the following tasks:

[2 pts] (A) Using the words **constant**, **linear**, **quadratic**, **cubic**, ***A***, ***B***, ***C***, and ***D***, complete the following phrases:

1) g_1 is the ________________ approximation for $f(x)$ at $x =$ _____
2) g_3 is the ________________ approximation for $f(x)$ at $x =$ _____
3) g_6 is the ________________ approximation for $f(x)$ at $x =$ _____
4) g_7 is the ________________ approximation for $f(x)$ at $x =$ _____

[1 pt] (B) Are the following statements **TRUE** (T) or **FALSE** (F) ?
[If printing test just circle T or F. If not printing test just copy the question number followed by T or F, like: `1) F`].

1) T / F : g_2 is the constant approximation for $f(x)$ at $x = A$.
2) T / F : g_2 is the linear approximation for $f(x)$ at $x = A$.
3) T / F : g_4 is the quadratic approximation for $f(x)$ at $x = D$.
4) T / F : g_5 is the linear approximation for $f(x)$ at $x = C$.



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4. **(3 pts) Taylor Series:**
[SDSU M151 S21 MiniTest4 V1 Q4 30/Apr/2021 5:00-6:10pm PDT — Do NOT share/distribute/post/upload]
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Compute the Taylor polynomial of order 2 (i.e. second degree polynomial) for $\boldsymbol{f(x) = A\cos(\omega x + \phi)}$ about $\boldsymbol{x = B}$, where $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{\omega}$, $\boldsymbol{\phi}$, are CONSTANTS.

$f(x) \approx$

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5. **(4 pts) Parametric Equations: Slopes and Tangents**

[`SDSU M151 S21 MiniTest4 V1 Q5 30/Apr/2021 5:00-6:10pm PDT` — Do NOT share/distribute/post/upload]

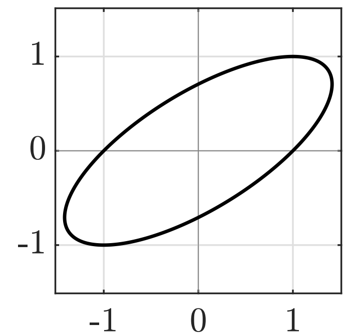
[Remember to **stop** solving and **submit** when there are **10 mins** (or more) left on the clock!!! No late submissions!]

Consider the following parametric equation for $\mathbf{0 \le t \le 2\pi}$:

$$\begin{cases} \boldsymbol{x(t) = \sin(t) + \cos(t)} \\ \boldsymbol{y(t) = \sin(t)} \end{cases}$$

whose graph is depicted in the figure.

[Hint: $\cos(\pi/4) = \sin(\pi/4) = \sqrt{2}/2$]



a) Using calculus, find ALL the points (x, y) that correspond to vertical AND horizontal tangency points. Show ALL your work... No work shown → no points!

Horizontal $t =$, $(x,y)=($, $)$ | $t =$, $(x,y)=($, $)$

Vertical: $t =$, $(x,y)=($, $)$ | $t =$, $(x,y)=($, $)$

b) For what value of t the parametric curve goes through the vertical axis ($x = 0$) for the FIRST time?

[Hint: check where the curve starts for $t = 0$ and in which direction it is being traced as t increases from $t = 0$.]

$t =$

Find the slope at this time.

$m =$

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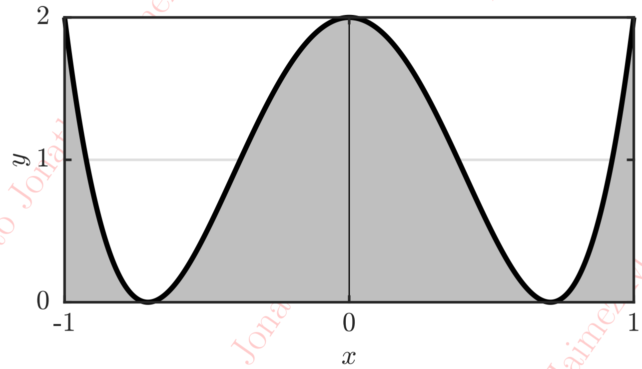
6. **(2.5 pts) Parametric Equations: Arclength and Areas**
[SDSU M151 S21 MiniTest4 V1 Q6 30/Apr/2021 5:00-6:10pm PDT — Do NOT share/distribute/post/upload]
[Remember to **stop** solving and **submit** when there are **10 mins** (or more) left on the clock!!! No late submissions!]

Consider the following parametric curve:

$$\begin{cases} x(t) = \sin(t) \\ y(t) = 1 + \cos(4t) \end{cases} \quad \text{for } -\frac{\pi}{2} \le t \le \frac{\pi}{2},$$

whose graph is depicted in the figure.



a) Write an explicit integral for the TOTAL length of the curve. [You do NOT need to compute the integral].

$$L = \int_{\square}^{\square} \boxed{} \, d\square$$

b) Write an explicit integral for the shaded area. [You do NOT need to compute the integral].

$$A = \int_{\square}^{\square} \boxed{} \, d\square$$

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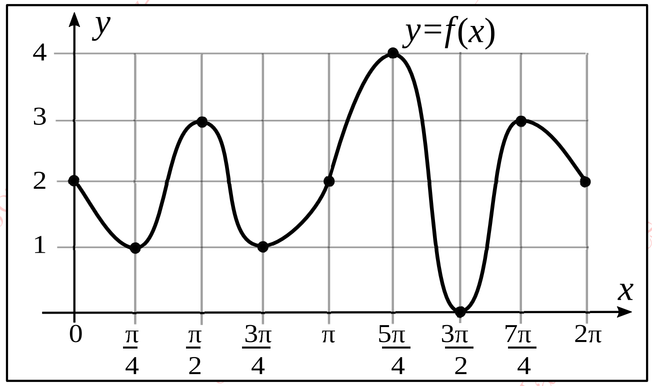
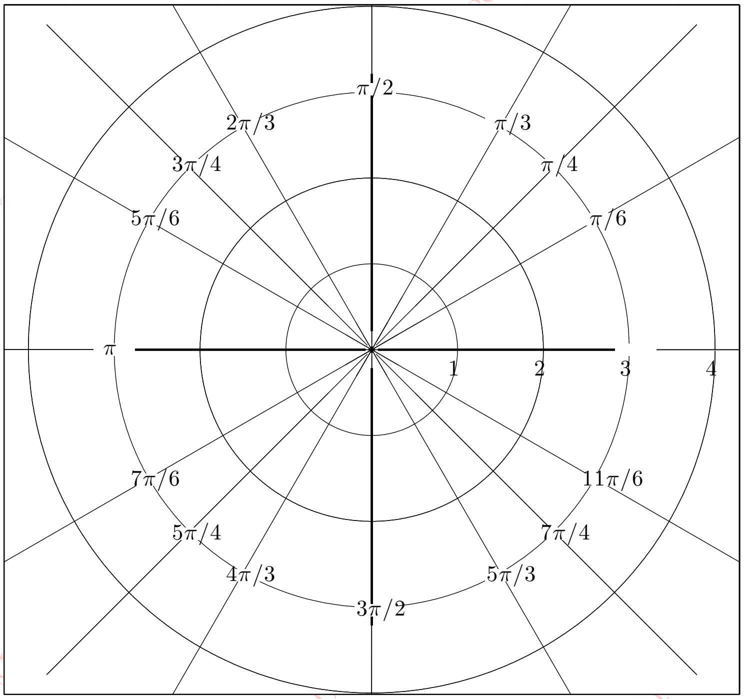
7. **(2.5 pts) Polar Equations:**
[SDSU M151 S21 MiniTest4 V1 Q7 30/Apr/2021 5:00-6:10pm PDT — Do NOT share/distribute/post/upload]
[Remember to **stop** solving and **submit** when there are **10 mins** (or more) left on the clock!!! No late submissions!]

The figure to the right represents the Cartesian $[(x, y)]$ plot of $y = f(x)$. Use this graph to sketch the **polar** curve $[(r, \theta)]$ traced by $r = f(\theta)$ for $0 \le \theta \le 2\pi$.





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8. **(3 pts) Polar Equations:**
[SDSU M151 S21 MiniTest4 V1 Q8 30/Apr/2021 5:00-6:10pm PDT — Do NOT share/distribute/post/upload]
[Remember to **stop** solving and **submit** when there are **10 mins** (or more) left on the clock!!! No late submissions!]

The figure to the right depicts the polar graph of $r = f(\theta)$.
In the same figure plot the following polar graphs:

a) $r = f(-\theta)$.

b) $r = f(\pi - \theta)$.

c) $r = \frac{1}{2} f(\theta)$.

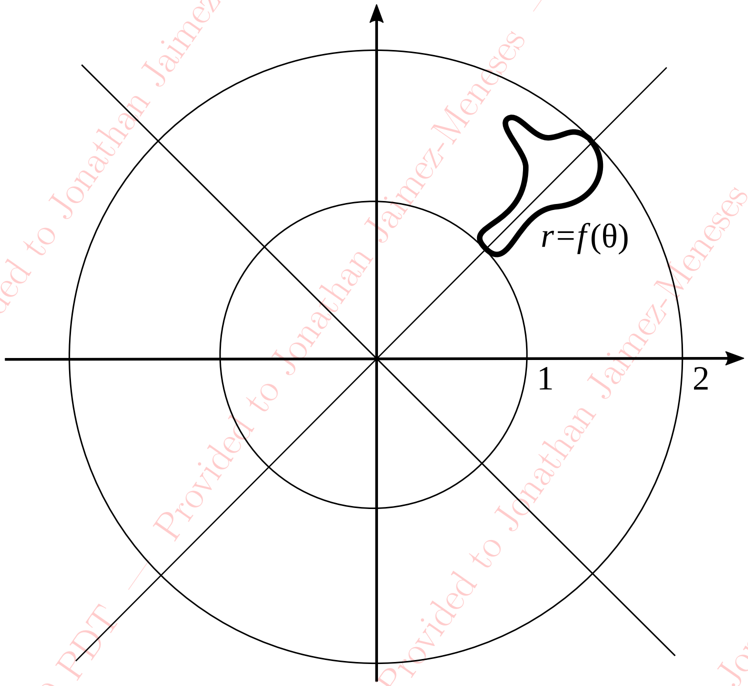
d) $r = f(\theta + \pi/4)$.

e) $r = f(\theta - \pi/4)$.

f) $r = -f(\theta)$.

Label each graph using '**a)**', '**b)**', ..., '**f)**'

IMPORTANT: If you do not label each graph you will earn NO points!



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9. **(2 pts) Parametric Equations: Areas and Lengths**
[SDSU M151 S21 MiniTest4 V1 Q9 30/Apr/2021 5:00-6:10pm PDT — Do NOT share/distribute/post/upload]
[Remember to **stop** solving and **submit** when there are **10 mins** (or more) left on the clock!!! No late submissions!]

[2 pts] Using calculus for PARAMETRIC curves:
Show that the perimeter of a circle of radius R is given by $\boldsymbol{L = 2\pi R}$.

[Extra credit] Using calculus for PARAMETRIC curves:
Show that the area of a disk of radius R is given by $\boldsymbol{A = \pi R^2}$.
[Hint#1: you might need the following integral: $\int \sin^2(x)\,dx = \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$.]
[Hint#2: if you go backwards then area is negative...]