Math 151 **Name / Section:** ____________________

FINAL Review

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1. Write an explicit integral for the volume V of the solid obtained by rotating the region bounded by the given curves about the specified line.

$$y = \frac{4}{9}x^2, y = \frac{13}{9} - x^2; \text{ about the } x\text{-axis}$$

2. Use the method of cylindrical shells to write an explicit integral for the volume V generated by rotating the region bounded by the given curves about the y-axis.

$$y = 12e^{-x^2}, y = 0, x = 0, x = 1$$

   Sketch the region and a typical shell.

3. Evaluate the integral. (Use C for the constant of integration. Assume m $\neq$ 0.)

$$\int t \sinh(mt)dt$$

4. Evaluate the integral. (Use C for the constant of integration.)

$$\int \sin^2(\pi x)\cos^5(\pi x)dx$$

5. Evaluate the integral. (Remember to use absolute values where appropriate. Use C for the constant of integration.)

$$\int \frac{3x^2 - 19x + 46}{(2x+1)(x-2)^2}dx$$

6. Evaluate the integral. (Use C for the constant of integration.)

$$\int x^3\sqrt{16 - x^2}\, dx$$

7. If the infinite curve $y = e^{-3x}, x \geq 0$ is rotated about the x-axis, find the surface area of the resulting surface.

8. The air in a room with volume 180 m^3 contains 0.15% carbon dioxide initially. Fresher air with only 0.05% carbon dioxide flows into the room at a rate of 2 m^3/min and the mixed air flows out at the same rate.

   Find the percentage of carbon dioxide in the room as a function of time t (in minutes).

   What happens with the percentage of carbon dioxide in the room in the long run?

9. Write TWO explicit integrals, one in x and one in y, for the length of the curve $y = \cos(x)$ for $0 \leq x \leq \pi$. DRAW A SKETCH!

10. Solve the initial value problem:

$$xy' = y + 3x^2 \sin x,\ \ y(\pi) = 0$$

11. Solve the initial value problem:

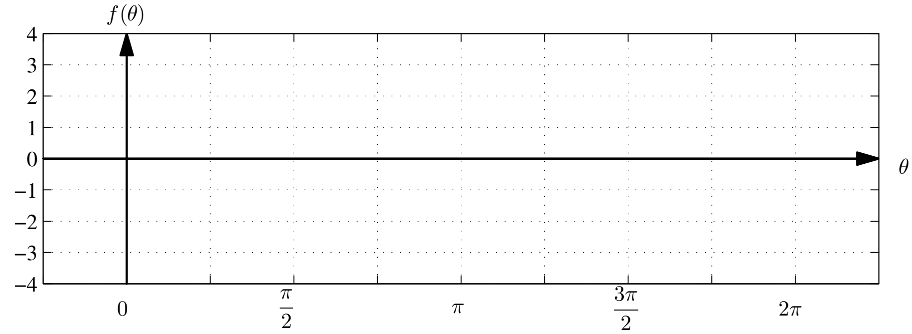
$$xy' = y^2,\ \ y(1) = 1$$

12. Find an equation ($y = ...$) of the tangent to the curve at the given point.

$$x = \cos t + \cos 2t,\ \ y = \sin t + \sin 2t,\ \ (x, y) = (-1, 1)$$

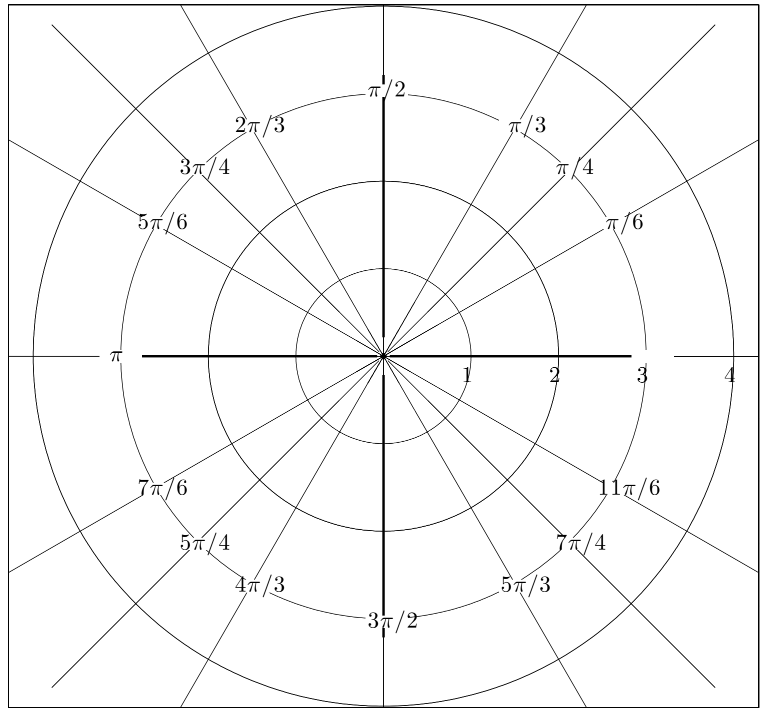
13. Find the Taylor series polynomial of order 3 for the function $y = \sinh(x)$ anchored at $x_0 = 3$.

14. Let $\boldsymbol{r = f(\theta) = 4\sin(\theta)}$



(A) Sketch the graph of $r = f(\theta)$ for $0 \le \theta \le 2\pi$ in CARTESIAN coordinates and identify ALL minima and maxima.

(B) Using (A) sketch the graph of $r = f(\theta)$ in POLAR.



(C)-(D): Use the fact that the slope in parametric is: $m = \dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$.

(C) Find all (r, θ) points where the curve as a HORIZONTAL tangent.

(D) Find all (r, θ) points where the curve as a VERTICAL tangent.

(E) By transforming coordinates from polar $\to$ Cartesian, find a CARTESIAN equation for this curve.

15. Determine the convergence of the following integral. If convergent, compute its value.
$$\int_0^3 \frac{2}{3-x}\,dx$$

16. Determine the convergence of the following integral. If convergent, compute its value.
$$\int_0^3 \frac{2}{(3-x)^2}\,dx$$

17. Determine the convergence of the following integral. If convergent, compute its value.
$$\int_4^\infty \frac{2}{(3-x)}\,dx$$

18. Find the radius of convergence and interval of convergence of the series.

(a) $\displaystyle\sum_{n=0}^{\infty} 4^n \frac{(x-2)^n}{n!}$

(b) $\displaystyle\sum_{n=0}^{\infty} (-1)^{n+1}\,(n^2+1)\,\frac{(x+5)^n}{3^n}$