Math 151 **Name / Section:** ____________________

Fall 2019

FINAL Review

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1. Write an explicit integral for the volume V of the solid obtained by rotating the region bounded by the given curves about the specified line.

$$y = \frac{4}{9}x^2, y = \frac{13}{9} - x^2; \text{ about the } x\text{-axis}$$

Answer:

$$V = \boxed{\pi \int_{-1}^{1} \left[\left(\frac{13}{9} - x^2 \right)^2 - \left(\frac{4}{9}x^2 \right)^2 \right] \mathrm{d}x}$$

2. Use the method of cylindrical shells to write an explicit integral for the volume V generated by rotating the region bounded by the given curves about the y-axis.

$$y = 12e^{-x^2}, y = 0, x = 0, x = 1$$

Sketch the region and a typical shell.

Answer:

$$V = \boxed{24\pi \int_{0}^{1} xe^{-x^2} \, \mathrm{d}x}$$

3. Evaluate the integral. (Use C for the constant of integration. Assume m $\neq$ 0.)

$$\int t \sinh(mt) \, \mathrm{d}t$$

Answer:

$$= \boxed{\frac{t}{m} \cosh(mt) - \frac{1}{m^2} \sinh(mt)}$$

4. Evaluate the integral. (Use C for the constant of integration.)

$$\int \sin^2(\pi x) \cos^5(\pi x) \, \mathrm{d}x$$

Answer:

$$= \boxed{\frac{1}{\pi} \left(\frac{1}{3} \sin^3(\pi x) - \frac{2}{5} \sin^5(\pi x) + \frac{1}{7} \sin^7(\pi x) \right) + C}$$

5. Evaluate the integral. (Remember to use absolute values where appropriate. Use C for the constant of integration.)

$$\int \frac{3x^2 - 19x + 46}{(2x+1)(x-2)^2} dx$$

Answer:

$$= \boxed{\frac{9}{2}\ln|2x+1| - 3\ln|x-2| - \frac{4}{x-2} + C}$$

6. Evaluate the integral. (Use C for the constant of integration.)

$$\int x^3\sqrt{16 - x^2}\, dx$$

Answer:

$$= \int 4^5 \sin\theta(1 - \cos^2\theta)\cos^2\theta \, \mathrm{d}\theta$$

$$= \frac{(16 - x^2)^{5/2}}{5} - \frac{16(16 - x^2)^{3/2}}{3}$$

7. If the infinite curve $y = e^{-3x}, x \geq 0$ is rotated about the x-axis, find the surface area of the resulting surface.

Answer:

$$= -\frac{2\pi}{9} \lim_{t\to\infty} \int_{\arctan(3)}^{\arctan(3e^{-3t})} \sec^3(\theta)\, \mathrm{d}\theta$$

$$= -\frac{2\pi}{9} \lim_{t\to\infty} \frac{1}{2}\left(\sec(\theta)\tan(\theta) + \ln|\sec(\theta) + \tan(\theta)|\right)\Big|_{\arctan(3)}^{\arctan(3e^{-3t})}$$

$$= \boxed{\frac{\pi}{9}\left((3\sqrt{10}) + \ln(\sqrt{10} + 3)\right)}$$

8. The air in a room with volume 180 m^3 contains 0.15% carbon dioxide initially. Fresher air with only 0.05% carbon dioxide flows into the room at a rate of 2 m^3/min and the mixed air flows out at the same rate.

Find the percentage of carbon dioxide in the room as a function of time t (in minutes).

What happens with the percentage of carbon dioxide in the room in the long run?

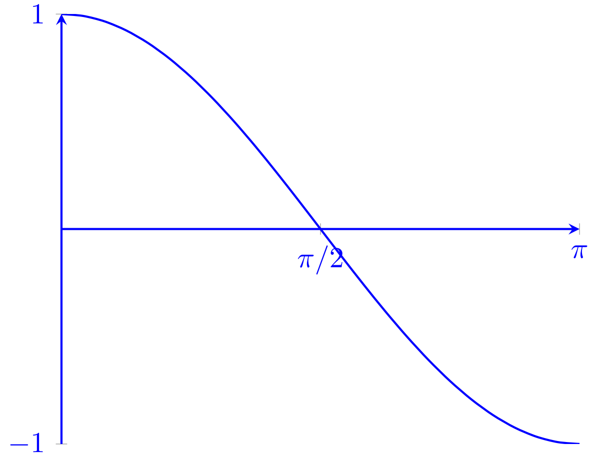
Answer: NOT READY YET

9. Write TWO explicit integrals, one in x and one in y, for the length of the curve $y = \cos(x)$ for $0 \leq x \leq \pi$. DRAW A SKETCH!

Answer:

$$= \boxed{\int_0^{\pi} \sqrt{1 + \sin^2(x)}\, \mathrm{d}x}$$

$$= \boxed{\int_{-1}^{1} \sqrt{1 + \frac{1}{1 - y^2}}\, \mathrm{d}y}$$



10. Solve the initial value problem:

$$xy' = y + 3x^2 \sin x,\ y(\pi) = 0$$

Answer:

$$y = -3x\cos x + Cx$$
$$C = -3$$

So the particular solution is $\boxed{y = -3x\cos x - 3x}$.

11. Solve the initial value problem:

$$xy' = y^2,\ y(1) = 1$$

Answer:

$$y = \frac{1}{C - \ln|x|}$$
$$y = \frac{1}{1 - \ln|x|}$$

12. Find an equation ($y = ...$) of the tangent to the curve at the given point.

$$x = \cos t + \cos 2t,\ y = \sin t + \sin 2t,\ (x, y) = (-1, 1)$$

Answer:

$$\Longrightarrow \boxed{y = 2x + 3}$$

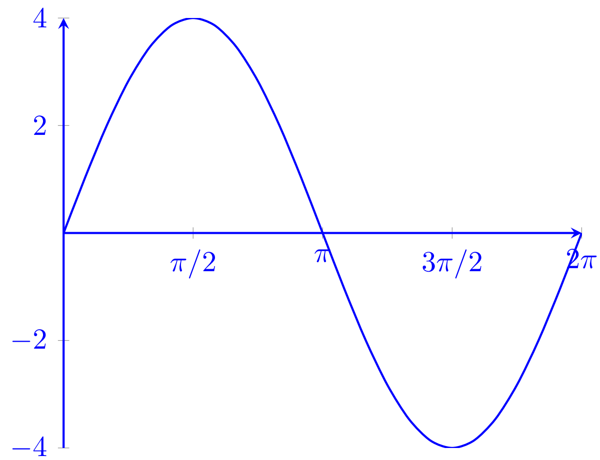
13. Find the Taylor series polynomial of order 3 for the function $y = \sinh(x)$ anchored at $x_0 = 3$.

Answer:

$$= \boxed{\sinh(3) + \cosh(3)(x-3) + \sinh(3)\frac{(x-3)^2}{2} + \cosh(3)\frac{(x-3)^3}{6}}$$

14. Let $\boldsymbol{r = f(\theta) = 4\sin(\theta)}$

(A) Sketch the graph of $r = f(\theta)$ for $0 \le \theta \le 2\pi$ in CARTESIAN coordinates and identify ALL minima and maxima.



Answer:
We can see from this that a maximum occurs at $\theta = \pi/2$ and a minimum occurs at $\theta = 3\pi/2$.

(B) Using (A) sketch the graph of $r = f(\theta)$ in POLAR.

Answer: The graph should be a circle of radius 2, repeating every π radians, centered at $(x, y) = (0, 2)$. The top, bottom, and sides of the circle should correspond to the horizontal and vertical tangents you find in the following steps.

(C)-(D): Use the fact that the slope in parametric is: $m = \dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$.

(C) Find all (r, θ) points where the curve as a HORIZONTAL tangent.

Answer: Since $r = 4\sin\theta$, then $x = 4\sin\theta\cos\theta$ and $y = 4\sin^2\theta$. Then we have that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{\mathrm{d}y}{\mathrm{d}\theta}}{\frac{\mathrm{d}x}{\mathrm{d}\theta}} = \frac{8\sin\theta\cos\theta}{4\cos^2\theta - 4\sin^2\theta}$$

$$\boxed{(0,0), (4,\pi/2), (0,\pi), (-4, 3\pi/2), (0, 2\pi)}$$

(D) Find all (r, θ) points where the curve as a VERTICAL tangent.

Answer:

$$\boxed{\left(\frac{\pi}{4}, \frac{4}{\sqrt{2}}\right), \left(\frac{3\pi}{4}, \frac{4}{\sqrt{2}}\right), \left(\frac{5\pi}{4}, -\frac{4}{\sqrt{2}}\right), \left(\frac{7\pi}{4}, -\frac{4}{\sqrt{2}}\right)}$$

(E) By transforming coordinates from polar $\to$ Cartesian, find a CARTESIAN equation for this curve.

Answer: This is a circle centered at $(0, 2)$ with radius 2, as we saw from the plot.

$$\boxed{x^2 + (y-2)^2 = 4}$$

15. Determine the convergence of the following integral. If convergent, compute its value.
$$\int_0^3 \frac{2}{3-x}\,dx$$
Answer:

$$\boxed{\text{Diverges}}$$

16. Determine the convergence of the following integral. If convergent, compute its value.
$$\int_0^3 \frac{2}{(3-x)^2}\,dx$$
Answer:

$$\boxed{\text{Diverges}}$$

17. Determine the convergence of the following integral. If convergent, compute its value.
$$\int_4^\infty \frac{2}{(3-x)}\,dx$$
Answer:

$$\boxed{\text{Diverges}}$$

18. Find the radius of convergence and interval of convergence of the series.

(a) $\displaystyle\sum_{n=0}^{\infty} 4^n \frac{(x-2)^n}{n!}$

Answer:

$$\boxed{\text{Radius of Convergence} = \infty}$$

$$\boxed{\text{Interval of Convergence} = (-\infty, \infty)}$$

(b) $\displaystyle\sum_{n=0}^{\infty} (-1)^{n+1} (n^2+1) \frac{(x+5)^n}{3^n}$

Answer:

$$\boxed{\text{Radius of Convergence} = 3}$$

$$\boxed{\text{Interval of Convergence} = (-8, -2)}$$