ACTIVITY#1 — Math 151 — Calculus II — Spring 2021

Professor/TA: _____ **Sec:** _____ **RedID:** _____

NAME (printed): _____ **Partners:** _____

(Family Name) (First Name)

**The sagging high voltage cables (hyperbolic functions)**

When laying high voltage transition cables a lot of attention needs to be taken care so that trees do not get close to them... or disasters happens: `http://carretero.sdsu.edu/teaching/M-151/labs/HangingCable.html`

The shape of a hanging cable is a *catenary* that is expressed through hyperbolic functions. The aim of this activity is to learn more about hyperbolic functions and their application to the above high voltage cable problem.
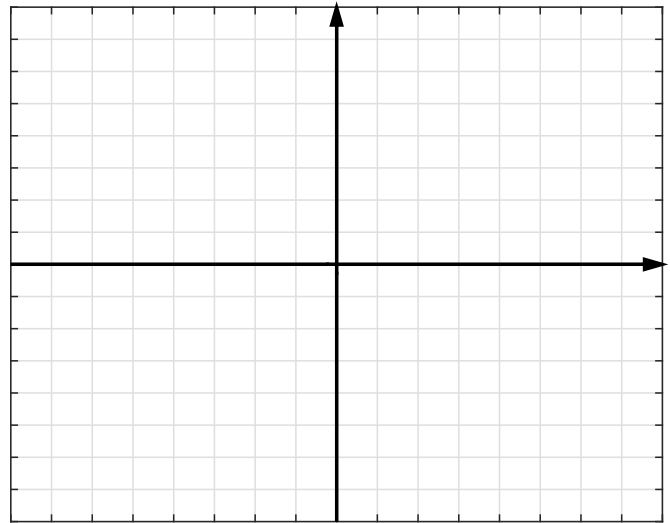
Let us sketch the graphs of the main hyperbolic functions: $\quad \cosh(x) = \dfrac{e^x + e^{-x}}{2} \quad$ and $\quad \sinh(x) = \dfrac{e^x - e^{-x}}{2}$.

(0) Using the definitions for sinh and cosh above, compute the limits when x tends to $-\infty$ and $+\infty$ for $\cosh(x)$ and $\sinh(x)$.

(1) Sketch the functions $f(x) = \frac{1}{2}e^x$ and $g(x) = \frac{1}{2}e^{-x}$.
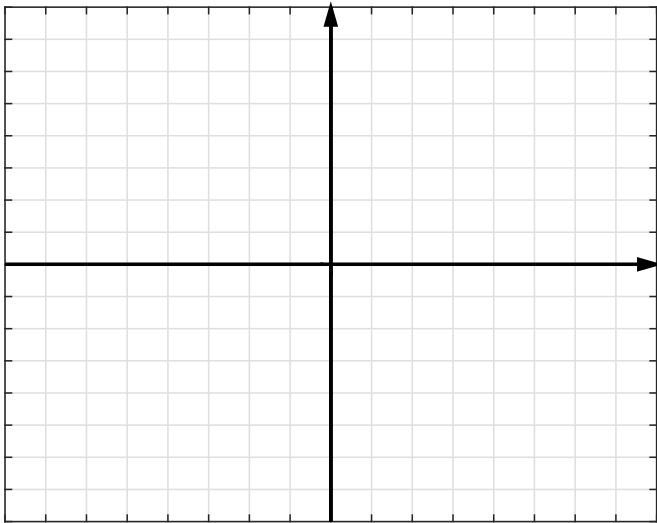
(2) Sketch the functions $f(x) = \frac{1}{2}e^x$ and $h(x) = -\frac{1}{2}e^{-x}$.

(3) On the same graph as (1) above sketch $\cosh(x)$. Also, on same graph as (2) above sketch $\sinh(x)$.

Are $\cosh(x)$ and $\sinh(x)$ odd or even functions or neither? Prove it mathematically! [Use the definitions with e^x and e^{-x}].

(4) The names of these hyperbolic functions are very similar to that of $\cos(x)$ and $\sin(x)$, which is no accident. Explore their similarities and differences by filling out the following tables. Your TA might prove one or two of these.

Function	Derivative
$\sin(x)$	
$\cos(x)$	
$\sinh(x)$	
$\cosh(x)$	

Identities
$\cos^2(x) + \sin^2(x) =$
Prove this one (use the definitions with e):
$\cosh^2(x) - \sinh^2(x) =$

(5) Application: The shape of a hanging cable between two same-height poles satisfies the equation $y = A\cosh(x/C) + B$. Assume that the cable's tension in our example is such that $A = 100$ and $C = 100$ and that the two poles of height h are placed at $x = -50$m and $x = +50$m as the graph shows. Then assume that the trees we are dealing with can reach a maximum height of 10m.

What pole height h is needed such that the cable just clears 10m?

Use the following value: $\cosh(0.5) \approx 1.13$ (no calculator needed!).

$$100\cosh\left(\frac{x}{100}\right) + B$$

h h 10m

−50m   0   50m
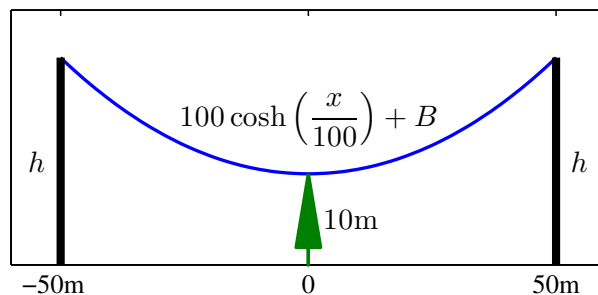
Bonus: Completed the work early? Calculate the length of cable for the above problem, using the following formula for cable length.
$$L(x) = \int_a^b \sqrt{1 + (f'(x))^2}\,dx,$$

where $f(x)$ is the hyperbolic function provided in the previous example: $f(x) = 100\cosh(x/100) + B$. Use the following value: $\sinh(0.5) \approx 0.52$ (no calculator needed!). Comment on your result by comparing to the amount of cable needed if it did not sag.