ACTIVITY#10 — Math 151 — Calculus II — Spring 2021

Professor/TA: ____________________ **Sec:** ______ **RedID:** ________________

NAME (printed): ________________ ________________ **Partners:** ____________________

(Family Name) **(First Name)**

SERIES: Let us see how series can be used in a wide range of applications.

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(1) Bouncy ball: A ball is dropped from a height of H meters. Under ideal conditions (no air resistance and no energy loss during bounces), after each bounce the ball returns to $1/3$ of its previous height.

(i) To what height does the ball rise after it hits the floor for the n-th time? How long does it take to completely stop?

(ii) Find the *total* vertical distance the ball travels before coming to rest.

(iii) Repeat the above if the ball returns to $1/b$ of its previous height with $b > 1$. Is the total length travelled finite or infinite?

(2) Retirement investment: Suppose an initial deposit of D dollars into a savings account yielding a fixed percentage p per year and that, at the end of each year, W dollars are withdrawn from the account. We would like to know how much capital a person needs to initially deposit when he/she retires to live for N years without investing any more capital and living off the W dollars withdrawn each year.

(i) Denote by A_n the amount in the savings account after n years. Thus, $A_0 = D$ corresponds to the initial deposit, A_1 to the amount after one year, A_2 to the amount after two years, etc...

(a) Find a recurrence relationship giving the next year's amount A_{n+1} as a function of the previous year's amount A_n.

(b) Write A_1, A_2, A_3 and, from there, find a direct formula giving A_N as a function of D, W, and p.

(ii) Under this saving strategy, how much money needs to be initially deposited to last exactly N years?
[Note: the partial sum of a geometric series is given by: $\sum_{i=1}^{N} b\, r^{i-1} = b\,(1 - r^N)/(1 - r)$].

(iii) In a realistic scenario, the average person retires at 65 and the life expectancy is about 80 years. Assume a 5% yield a year and that the person needs $30,000$ per year to sustain themselves. How much does this person need to invest at retirement?

(iv) Compare this to the total amount needed to live for 15 years at $30,000$ per year ($= 15 \times 30,000 = 450,000$).