ACTIVITY#12 — Math 151 — Calculus II — Spring 2021

Professor/TA: ________________ **Sec:** __________ **RedID:** __________

NAME (printed): ____________ ____________ **Partners:** ________________

(Family Name) **(First Name)**

Parametric equations: application to Bézier curves and CAD. Bézier curves are used in computer-aided design (CAD) and are named after the French Pierre Bézier (1910-1999), who worked in the automotive industry.

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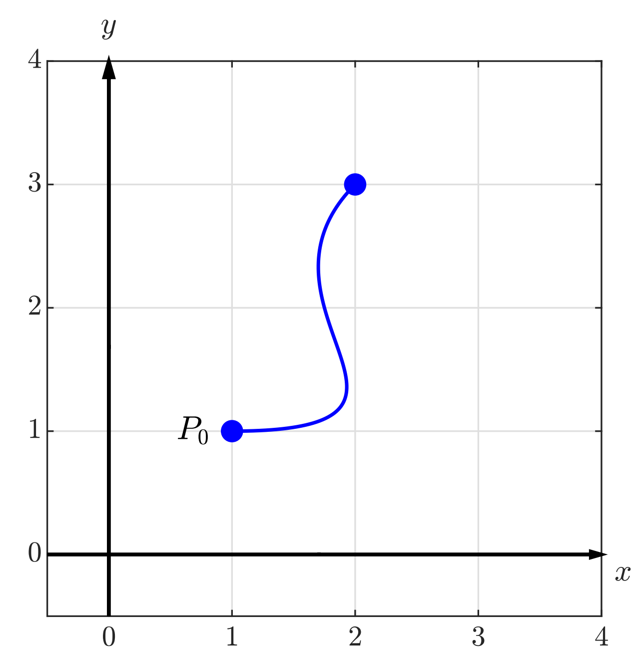
(1) A *cubic* Bézier curve is determined by four points $P_0(x_0, y_0)$, $P_1(x_1, y_1)$, $P_2(x_2, y_2)$, and $P_3(x_3, y_3)$, and is defined by the *parametric equation*:

$$P(t): \begin{cases} x(t) = f(t) = x_0\,(1-t)^3 + 3x_1\,t\,(1-t)^2 + 3x_2\,t^2\,(1-t) + x_3\,t^3 \\ y(t) = g(t) = y_0\,(1-t)^3 + 3y_1\,t\,(1-t)^2 + 3y_2\,t^2\,(1-t) + y_3\,t^3 \end{cases} \tag{1}$$

for $0 \le t \le 1$. First let us understand what the four points $P_i(x_i, y_i)$ represent graphically.

(a) What points do you recover when $t = 0$ and $t = 1$?

(b) Using the formula for the slope of a parametric curve (you will learn this in class very soon): $m = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ compute the slopes m_0 and m_3 at the points, respectively, $P(t = 0)$ and $P(t = 1)$ found above.
Simplify as much as possible and interpret these slopes with respect to the position of the points P_i.



(c) Use the information you obtained above to locate the possible locations for the points P_1, P_2, and P_3 on the typical Bézier curve sketch to the right. Make sure to match the tangent lines at the end points of the curve with the location of the points P_i.

(2) Let us "play" a bit with Bézier curves.

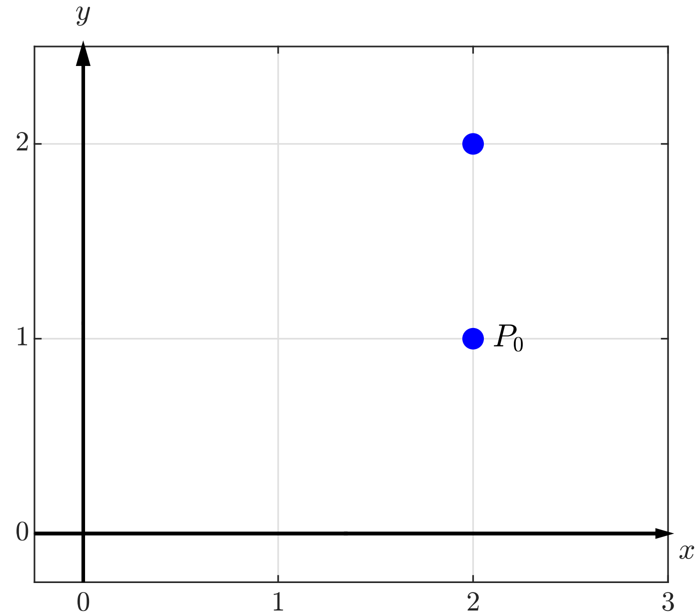
(a) Use the following `desmos` code to play around with the curves you get by varying the points P_i. Note in particular the slope that the points P_1 and P_2 allow you to define. Desmos code: `https://www.desmos.com/calculator/rhkn2fzb1a`

(b) Let us draw, using a single Bézier curve, a letter `C` such that:

(i) the lower end (P_0) coincides with the point (2,1),
(ii) it has height one, and
(iii) the tangent lines at the end points are horizontal.

Find the coordinates of the four points P_i that generate such letter. Draw a sketch of this letter `C` and the points and the slopes at P_0 and P_3.

Could you get a 'nicer' `C` by modifying the slopes of the tangent lines? Overlay the sketch of your improved letter `C` in the sketch to the right.

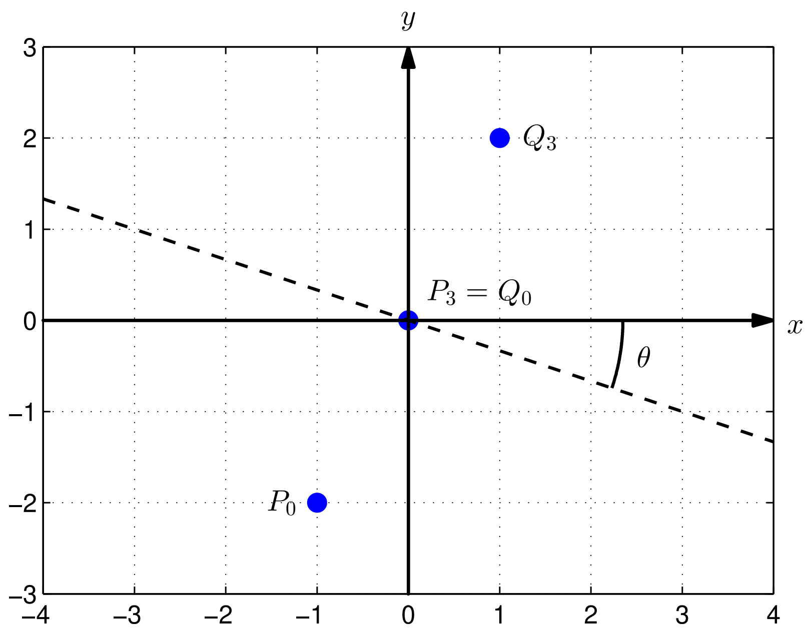


(c) Let us now concatenate two Bézier curves to draw a letter `S` such that:

(i) it starts at $(-1,-2)$ and ends at $(1,2)$,
(ii) passes through the origin,
(iii) it is symmetric with respect to the origin,
(iv) has an inflection point at the origin with a tangent line making an angle of θ as depicted.

To do this let us define TWO Bézier curves with points (P_0, P_1, P_2, P_3) and (Q_0, Q_1, Q_2, Q_3).

What condition(s) on the points P_2 and Q_1 need to be satisfied so that the letter is *smooth* at *all* points?



Fill the blanks for all the points' the coordinates [where a and θ (see angle in the figure) are positive c constants that slightly change the shape details of the letter `S`]:

$$P_0 \begin{pmatrix} -1 \\ -2 \end{pmatrix}, \quad P_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix}, \quad P_2 \begin{pmatrix} a \\ \boxed{} \end{pmatrix}, \quad P_3 \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$Q_0 \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad Q_1 \begin{pmatrix} \boxed{} \\ \boxed{} \end{pmatrix}, \quad Q_2 \begin{pmatrix} \boxed{} \\ \boxed{} \end{pmatrix}, \quad Q_3 \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

Draw *all* the points and associated tangent lines in the sketch.
Desmos code: `https://www.desmos.com/calculator/njc1cqobpx`