ACTIVITY#13 — Math 151 — Calculus II — Spring 2021

Professor/TA: ______________ **Sec:** ______________ **RedID:** ______________

NAME (printed): ______________ ______________ **Partners:** ______________

(Family Name) **(First Name)**

Calculus on PARAMETRIC and POLAR eqns. A little birdy told me that the final includes similar problems!!!

(1) PARAMETRIC: Let's do some calc: Consider the curve $\mathcal{P}$ defined by: $\begin{cases} x = t^2 \\ y = t^3 - 3t \end{cases}$ for $-2 \le t \le 2$.

(i) Show that $\mathcal{P}$ has two tangents at the point $(3, 0)$ and find their slopes.

(ii) Find the points on $\mathcal{P}$ where the tangent is horizontal.

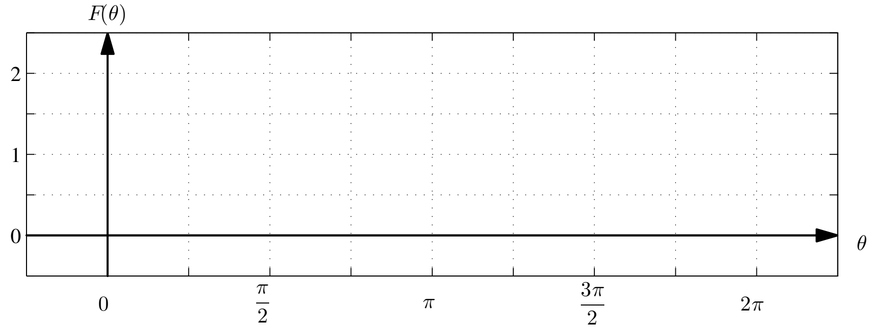
(iii) Find the points on $\mathcal{P}$ where the tangent is vertical.

(iv) Using the above information and using the starting and finishing points at $t = -2$ and $t = +2$, respectively, sketch the curve and the tangent lines. Use arrows to indicate the direction the curve is traced.

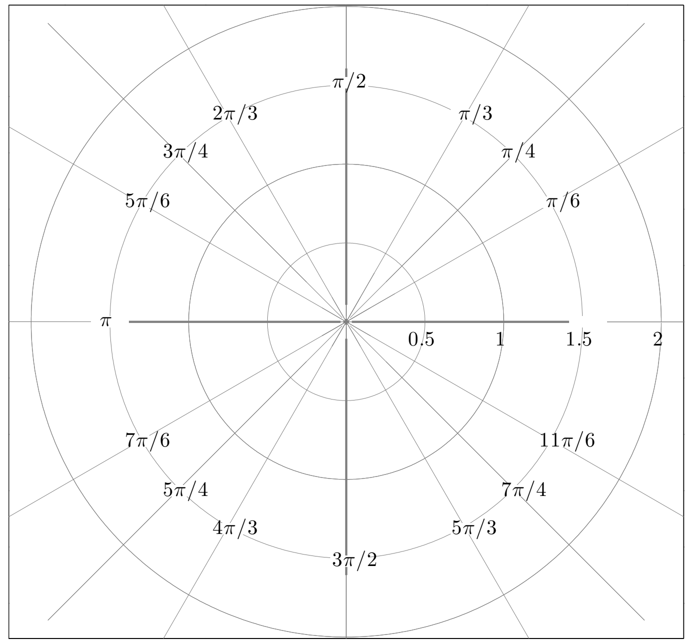
(2) POLAR: Consider the curve $\mathcal{C}$ defined by:

$$\boldsymbol{r = F(\theta) = 1 - \sin\theta}.$$

(i) Sketch the graph of $F(\theta)$ for $0 \le \theta \le 2\pi$ in Cartesian coordinates. Clearly indicate the points where (a) $F(\theta) = 0$, and the points at which F attains a local (b) maximum or (c) minimum.



(ii) By using the results you obtained in the previous point, sketch $\mathcal{C}$ in **POLAR** coordinates for $0 \le \theta \le 2\pi$.



(iii) Write $\mathcal{C}$ in parametric form by using:

$$\begin{cases} \boldsymbol{x = r\,\cos(\theta) = F(\theta)\,\cos(\theta)} \\ \boldsymbol{y = r\,\sin(\theta) \;= F(\theta)\,\sin(\theta)} \end{cases}$$

From this perform the following tasks.
You'll need the slope in parametric: $m(\theta) = \dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$.

(a) Find ALL horizontal points of tangency.

(b) Find ALL vertical points of tangency.

(c) Sketch all these tangents in the polar plot. [You might use this to enhance your graph!]. Do you see the ♡? This curve is called a cardioid (from the Greek word "καρδια" meaning heart).