

# Cheat Sheet for Midterm#1 - M252 - Calculus III - Fall 2022

This cheat sheet will be included in the midterm. You do NOT have to memorize these formulas. However, make sure that you understand and know how to apply ALL of them!!!

- If  $\vec{r} = \langle x, y, z \rangle$  then its norm (magnitude) is  $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$
- Making a vector unitary:  $\hat{\mathbf{a}} = \frac{\vec{\mathbf{a}}}{|\vec{\mathbf{a}}|}$      $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = |\vec{\mathbf{a}}| |\vec{\mathbf{b}}| \cos(\theta)$      $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 0 \Leftrightarrow \vec{\mathbf{a}} \perp \vec{\mathbf{b}}$      $\text{proj}_{\vec{\mathbf{a}}}(\vec{\mathbf{b}}) = \frac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{|\vec{\mathbf{a}}|} \hat{\mathbf{a}}$ .
- $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = |3 \times 3 \text{ determinant}|$  (sign - for middle entry!)     $|\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = |\vec{\mathbf{a}}| |\vec{\mathbf{b}}| \sin(\theta)$
- $|\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = \text{area of parallelogram} (= 2 \times \text{area of triangle})$      $|\vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}})| = \text{volume of parallelepiped}$ .
- Tangent vector:  $\vec{\mathbf{T}} = \vec{\mathbf{r}}'(t)$ , unitary:  $\hat{\mathbf{T}} = \frac{\vec{\mathbf{r}}'(t)}{|\vec{\mathbf{r}}'(t)|}$ .
- If  $|\vec{\mathbf{r}}'(t)| = \text{const}$  then  $\vec{\mathbf{r}}' \perp \vec{\mathbf{r}}$ .
- Arclength:  $L = \int_{t_1}^{t_2} |\vec{\mathbf{r}}'(t)| dt = \int_{t_1}^{t_2} \sqrt{f'^2 + g'^2 + h'^2} dt$      $s(t) = \int_{t_1}^t |\vec{\mathbf{r}}'(\tau)| d\tau$ .
- Curvature in 3D:  $\kappa = 1/\text{radius} = \left| \frac{d\hat{\mathbf{T}}}{ds} \right| = \left| \frac{\hat{\mathbf{T}}'}{|\vec{\mathbf{r}}'|} \right| = \frac{|\vec{\mathbf{r}}' \times \vec{\mathbf{r}}''|}{|\vec{\mathbf{r}}'|^3}$ .
- Curvature for planar curve  $y = F(x)$  in 2D:  $\kappa = \frac{|F''|}{(1 + F'^2)^{3/2}}$ .
- Normal & binormal:  $\hat{\mathbf{N}} = \frac{\hat{\mathbf{T}}'}{|\hat{\mathbf{T}}'|}$      $\hat{\mathbf{B}} = \hat{\mathbf{T}} \times \hat{\mathbf{N}}$ .
- $\vec{\mathbf{v}}(t) = \int \vec{\mathbf{a}}(t) + \vec{\mathbf{v}}(0)$      $\vec{\mathbf{r}}(t) = \int \vec{\mathbf{v}}(t) + \vec{\mathbf{r}}(0)$      $\vec{\mathbf{a}}(t) = \vec{\mathbf{v}}'(t) = \vec{\mathbf{r}}''(t)$ .
- Tangential and normal components of the acceleration:  
 $\vec{\mathbf{a}} = v' \hat{\mathbf{T}} + \kappa v^2 \hat{\mathbf{N}} = a_T \hat{\mathbf{T}} + a_N \hat{\mathbf{N}}$      $v = |\vec{\mathbf{r}}'|$      $a_T = \frac{|\vec{\mathbf{r}}' \cdot \vec{\mathbf{r}}''|}{|\vec{\mathbf{r}}'|}$      $a_N = \frac{|\vec{\mathbf{r}}' \times \vec{\mathbf{r}}''|}{|\vec{\mathbf{r}}'|}$