Cheat Sheet for Midterm#2 - M252 - Calculus III - Fall 2022

This cheat sheet will be included in the midterm. You do NOT have to memorize these formulas. However, make sure that you understand and know how to apply ALL of them!!!

- When dealing with functions of several variables, be careful about which variable you are doing the derivative (or integral) with respect to! [Remember the: "Integral (or derivative) with respect to what?"]
- Clairant's theorem: If f, f_{xy}, and f_{yx} are continuous $\Rightarrow$ $f_{xy} = f_{yx}$.
- Tangent plane to $f(x,y)$ at $z_0 = f(x_0,y_0)$: $z - z_0 = f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$.
  $\Rightarrow$ Linear approximation: $z = z_0 + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$.
  $\Rightarrow$ Differentials: $dz = f_x(x_0,y_0)\,dx + f_y(x_0,y_0)\,dy$
- Chain rule (case 1): $f = f(x,y)$ and $x = x(t)$ and $y = y(t)$: $\dfrac{df}{dt} = \dfrac{dx}{dt}\dfrac{\partial f}{\partial x} + \dfrac{dy}{dt}\dfrac{\partial f}{\partial y}$.
- Chain rule (case 2): $f = f(x,y)$ and $x = g(s,t)$ and $y = h(s,t)$:
  $\dfrac{df}{ds} = \dfrac{\partial f}{\partial x}\dfrac{\partial x}{\partial s} + \dfrac{\partial f}{\partial y}\dfrac{\partial y}{\partial s}$ and $\dfrac{df}{dt} = \dfrac{\partial f}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial f}{\partial y}\dfrac{\partial y}{\partial t}$
- Implicit differentiation: $F(x,y) = 0$: $\dfrac{dy}{dx} = -\dfrac{F_x}{F_y}$ ‖ $F(x,y,z) = 0$: $\dfrac{dz}{dx} = -\dfrac{F_x}{F_z}$ & $\dfrac{dz}{dy} = -\dfrac{F_y}{F_z}$.
- Directional derivative along $\widehat{\boldsymbol{u}} = \langle a, b\rangle$: $D_{\widehat{\boldsymbol{u}}} f(x,y) = f_x(x,y)\,a + f_y(x,y)\,b = \nabla f \cdot \widehat{\boldsymbol{u}}$.
- Gradient: Nabla operator $= \nabla = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\rangle$ ‖ gradient$(f) = \nabla f(x,y,z) = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\rangle f = \langle f_x, f_y, f_z\rangle$.
- Direction of steepest ASCENT is given by gradient $\nabla f(x,y,z)$ ‖ Gradient vector $\perp$ level curves.
- Tangent plane to $F(x,y,z) = k$ at (x_0,y_0,z_0): $(x-x_0)\,F_x + (y-y_0)\,F_y + (z-z_0)\,F_z = 0$ [with partials evaluated at (x_0,y_0,z_0)].
- Normal line to $F(x,y,z) = k$ at (x_0,y_0,z_0): $\dfrac{x-x_0}{F_x} = \dfrac{y-y_0}{F_y} = \dfrac{z-z_0}{F_z}$ [with partials evaluated at (x_0,y_0,z_0)].
- Remember difference between local and global max/min ‖ For max/min also check along boundaries!
- Max/min will be at critical points: $f_x = 0 = f_y$ (also check boundaries!)
- Second derivative test: $f_x = 0 = f_y$ and Hessian: $D(x,y) = f_{xx}f_{yy} - (f_{xy})^2 = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$:
  (a) $D > 0$ & $f_{xx} > 0 \Rightarrow$ local min ‖ (b) $D > 0$ & $f_{xx} < 0 \Rightarrow$ local max ‖ (c) $D < 0 \Rightarrow$ saddle.
- Lagrange multiplier: min/max of f with constraint $g = k$: $\nabla f(x,y,z) = \lambda \nabla g(x,y,z)$ & $g(x,y,z) = k$.
- Fubini's theorem: if domain is rectangular $\Rightarrow \int_{x=a}^{b}\int_{y=c}^{d} f\,dy\,dx = \int_{y=c}^{d}\int_{x=a}^{b} f\,dx\,dy$.
- Average: $\bar{f} = \frac{1}{A(\mathcal{D})}\iint_{\mathcal{D}} f(x,y)\,dA$.
- Area using double integral: Area$(\mathcal{D})$=$\iint_{\mathcal{D}} 1\,dA$.
- Type I: $\iint_{\mathcal{D}} f(x,y)\,dA = \int_{x=a}^{b}\int_{y=g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$.
- Type II: $\iint_{\mathcal{D}} f(x,y)\,dA = \int_{y=c}^{d}\int_{x=h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy$.
- Polar: $\iint_{\mathcal{D}} f(x,y)\,dA = \iint_{\mathcal{D}} f(r\cos(\theta), r\sin(\theta))\,\boxed{r}\,dr\,d\theta$. ‖ $r^2 = x^2 + y^2$, $x = r\cos(\theta)$, $y = r\sin(\theta)$.
- Trigonometric identities/formulas:
  - $\sin(x \pm y) = \sin x\,\cos y \pm \cos x\,\sin y$
  - $\cos(x \pm y) = \cos x\,\cos y \mp \sin x\,\sin y$
  - $\sin 2x = 2\sin x\,\cos x$
  - $\cos 2x = 1 - 2\sin^2 x$
  - $\sin^2 x = \dfrac{1 - \cos 2x}{2}$
  - $\cos^2 x = \dfrac{1 + \cos 2x}{2}$
  - $\sin A\,\cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
  - $\sin A\,\sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
  - $\cos A\,\cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$
  - $\sinh(x) = \frac{1}{2}[e^x - e^{-x}]$
  - $\cosh(x) = \frac{1}{2}[e^x + e^{-x}]$