Must Know Material for Midterm#2 - M252 - Calculus III - Fall 2022

List of the material that MUST be second nature to you in preparation for MT#2:

- **Chap. 14 Partial Derivatives:**
  14.1: Functions of Several Variables
  14.2: Limits and Continuity
  14.3: Partial Derivatives
  14.4: Tangent Planes and Linear Approximations
  14.5: The Chain Rule
  14.6: Directional Derivatives & Gradient Vector
  14.7: Maximum and Minimum Values
  14.8: Lagrange Multipliers
- **Chap. 15 Multiple Integrals:**
  15.1: Double Integrals over Rectangles
  15.2: Double Integrals over General Regions
  15.3: Double Integrals in Polar Coordinates

You must be very confident with the following basic and fundamental topics/formulas/techniques:

- Need to know the material about planes and lines from MT#1.
- If a constant is not defined (like an α or any other symbol), do NOT assign a value for it. It is a fixed scalar (or vector) and you should just leave it as it is.
- When dealing with functions of several variables, be careful about which variable you are doing the derivative (or integral) with respect to! [Remember the: "Integral (or derivative) with respect to what?"]
- Clairant's theorem: If f, f_{xy}, and f_{yx} are continuous $\Rightarrow$ $f_{xy} = f_{yx}$.
- Tangent plane to $f(x, y)$ at $z_0 = f(x_0, y_0)$: $z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$.
  $\Rightarrow$ Linear approximation: $z = z_0 + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$.
  $\Rightarrow$ Differentials: $dz = f_x(x_0, y_0)\, dx + f_y(x_0, y_0)\, dy$
- Chain rule (case 1): $f = f(x, y)$ and $x = x(t)$ and $y = y(t)$: $\dfrac{df}{dt} = \dfrac{dx}{dt}\dfrac{\partial f}{\partial x} + \dfrac{dy}{dt}\dfrac{\partial f}{\partial y}$.
- Chain rule (case 2): $f = f(x, y)$ and $x = g(s, t)$ and $y = h(s, t)$:
  $\dfrac{df}{ds} = \dfrac{\partial f}{\partial x}\dfrac{\partial x}{\partial s} + \dfrac{\partial f}{\partial y}\dfrac{\partial y}{\partial s}$ and $\dfrac{df}{dt} = \dfrac{\partial f}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial f}{\partial y}\dfrac{\partial y}{\partial t}$
- Implicit differentiation: $F(x, y) = 0$: $\dfrac{dy}{dx} = -\dfrac{F_x}{F_y}$ ‖ $F(x, y, z) = 0$: $\dfrac{dz}{dx} = -\dfrac{F_x}{F_z}$ & $\dfrac{dz}{dy} = -\dfrac{F_y}{F_z}$.
- Directional derivative along $\hat{\boldsymbol{u}} = \langle a, b\rangle$: $D_{\hat{\boldsymbol{u}}} f(x, y) = f_x(x, y)\, a + f_y(x, y)\, b = \overrightarrow{\nabla f} \cdot \hat{\boldsymbol{u}}$.
- Gradient: Nabla operator $= \nabla = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\rangle$ ‖ gradient$(f) = \overrightarrow{\nabla f}(x, y, z) = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\rangle f = \langle f_x, f_y, f_z\rangle$.
- Direction of steepest ASCENT is given by gradient $\overrightarrow{\nabla f}(x, y, z)$ ‖ Gradient vector $\perp$ level curves.
- Tangent plane to $F(x, y, z) = k$ at (x_0, y_0, z_0): $(x - x_0)\, F_x + (y - y_0)\, F_y + (z - z_0)\, F_z = 0$ [with partials evaluated at (x_0, y_0, z_0)].
- Normal line to $F(x, y, z) = k$ at (x_0, y_0, z_0): $\dfrac{x - x_0}{F_x} = \dfrac{y - y_0}{F_y} = \dfrac{z - z_0}{F_z}$ [with partials evaluated at (x_0, y_0, z_0)].
- Remember difference between local and global max/min ‖ For max/min also check along boundaries!
- Max/min will be at critical points: $f_x = 0 = f_y$ (also check boundaries!)
- Second derivative test: $f_x = 0 = f_y$ and Hessian: $D(x, y) = f_{xx} f_{yy} - (f_{xy})^2 = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$:
  (a) $D > 0$ & $f_{xx} > 0 \Rightarrow$ local min ‖ (b) $D > 0$ & $f_{xx} < 0 \Rightarrow$ local max ‖ (c) $D < 0 \Rightarrow$ saddle.
- Lagrange multiplier: min/max of f with constraint $g = k$: $\overrightarrow{\nabla f}(x, y, z) = \lambda \overrightarrow{\nabla g}(x, y, z)$ & $g(x, y, z) = k$.
- Fubini's theorem: if domain is rectangular $\Rightarrow \int_{x=a}^{b} \int_{y=c}^{d} f\, dy\, dx = \int_{y=c}^{d} \int_{x=a}^{b} f\, dx\, dy$.
- Average: $\bar{f} = \frac{1}{A(\mathcal{D})} \iint_{\mathcal{D}} f(x, y)\, dA$.
- Area using double integral: Area$(\mathcal{D})$=$\iint_{\mathcal{D}} 1\, dA$.
- Type I: $\iint_{\mathcal{D}} f(x, y)\, dA = \int_{x=a}^{b} \int_{y=g_1(x)}^{g_2(x)} f(x, y)\, dy\, dx$.
- Type II: $\iint_{\mathcal{D}} f(x, y)\, dA = \int_{y=c}^{d} \int_{x=h_1(y)}^{h_2(y)} f(x, y)\, dx\, dy$.
- Polar: $\iint_{\mathcal{D}} f(x, y)\, dA = \iint_{\mathcal{D}} f(r\cos(\theta), r\sin(\theta))\, \boxed{r}\, dr\, d\theta$. ‖ $r^2 = x^2 + y^2$, $x = r\cos(\theta)$, $y = r\sin(\theta)$.
- Make sure that you understand and know how to apply ALL the formulas given in the cheat sheet!