Cheat Sheet for Midterm#3 - M252 - Calculus III - Fall 2022

This cheat sheet will be included in the midterm. You do NOT have to memorize these formulas. However, make sure that you understand and know how to apply ALL of them!!!

- If a constant is not defined (like an α or any other symbol), do NOT assign a value for it. It is a fixed scalar (or vector) and you should just leave it as it is.
- When dealing with functions of several variables, be careful about which variable you are doing the derivative (or integral) with respect to! [Remember the: "Integral (or derivative) with respect to what?"]
- Gradient: Nabla operator $= \nabla = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle$ ‖ gradient$(f) = \overrightarrow{\nabla} f(x,y,z) = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle f = \langle f_x, f_y, f_z \rangle$.
- Direction of steepest ASCENT is given by gradient $\nabla f(x,y,z)$ ‖ Gradient vector $\perp$ level curves.
- Polar: $\iint_{\mathcal{D}} f(x,y)\,dA = \iint_{\mathcal{D}} f(r\cos(\theta), r\sin(\theta))\,\boxed{r}\,dr\,d\theta$. ‖ $r^2 = x^2 + y^2$, $x = r\cos(\theta)$, $y = r\sin(\theta)$.
- Cylindrical: $\iiint_{\mathcal{B}} f(x,y,z)\,dV = \iiint_{\mathcal{B}} f(r\cos(\theta), r\sin(\theta), z)\,\boxed{r}\,dr\,d\theta\,dz$.
  with $r^2 = x^2 + y^2 + z^2$, $x = r\cos(\theta)$, $y = r\sin(\theta)$, $z = z$.
- Spherical: $\iiint_{\mathcal{B}} f(x,y,z)\,dV = \iiint_{\mathcal{B}} f(\rho\sin(\phi)\cos(\theta), \rho\sin(\phi)\sin(\theta), \rho\cos(\phi))\,\boxed{\rho^2\sin(\phi)}\,d\rho\,d\theta\,d\phi$.
  with $\rho^2 = x^2 + y^2 + z^2$, $x = \rho\sin(\phi)\cos(\theta)$, $y = \rho\sin(\phi)\sin(\theta)$, $z = \rho\cos(\phi)$.
- Type I, type II, and type III for triple integrals.
- Surface area: $S = \iint_{\mathcal{D}} \sqrt{f_x^2 + f_y^2 + 1}\,dA$.
- Volume: $V(\mathcal{B}) = \iiint_{\mathcal{B}} dV$
- Mass: $M(\mathcal{B}) = \iiint_{\mathcal{B}} \rho(x,y,z)\,dV$ where $\rho =$ density.
- Change of variables $(x,y,z) \to (u,v,w)$: $\iiint f(x,y,z)\,dV = \iiint f(u,v,w)\,|J|\,du\,dv\,dw$
  where $J = \left| \frac{\partial(x,y,z)}{\partial(u,v,w)} \right|$ is the Jacobian of the transformation. [Similar for double integrals]
- Line integrals for *scalar* fields:
  ○ 2D: $\int_{\mathcal{C}} f(x,y)\,dS = \int_{t=a}^{b} f(x(t), y(t))\sqrt{(dx/dt)^2 + (dy/dt)^2}\,dt$ ‖ $x = x(t)$ & $y = y(t)$ for $a \le t \le b$
  ○ 3D (and 2D): $\int_{\mathcal{C}} f(x,y,z)\,dS = \int_{t=a}^{b} f(\vec{\boldsymbol{r}}(t))\,|\vec{\boldsymbol{r}}\,'(t)|\,dt$ ‖ $\vec{\boldsymbol{r}}(t) = \langle x(t), y(t), z(t)\rangle$ for $a \le t \le b$
- Line integrals for *vector* fields: $\int_{\mathcal{C}} \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}} = \int_{\mathcal{C}} \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{T}}\,dS = \int_{t=a}^{b} \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{r}}\,'\,dt$
- Work: $\vec{\boldsymbol{F}}(x,y,z) = \langle P(x,y,z), Q(x,y,z), R(x,y,z)\rangle$: $W = \int_{\mathcal{C}} \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}} = \int_{t=a}^{b} \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{r}}\,'\,dt = \int P\,dx + Q\,dy + R\,dz$
- Fundamental theorem: $\int_{\mathcal{C}} \overrightarrow{\nabla} f \cdot d\vec{\boldsymbol{r}} = f(\vec{\boldsymbol{r}}(b)) - f(\vec{\boldsymbol{r}}(a))$
- A vector field is conservative if it comes from the gradient of a scalar field.
- $\vec{\boldsymbol{F}}(x,y) = \langle P(x,y), Q(x,y)\rangle$ conservative $\Leftrightarrow$ $P_y = Q_x$ [if on a simply-connected domain].
- Green's theorem (GT): $\oint_{\mathcal{C}} P\,dx + Q\,dy = \iint_{\mathcal{D}} (Q_x - P_y)\,dA$ [path must be oriented *anti-clockwise*].
- Punching holes: *inner* paths must be oriented *clockwise* [i.e., *opposite*]. Remember: $\oint_{-\mathcal{C}} = -\oint_{\mathcal{C}}$
- Area using GT: Area enclosed by $\mathcal{C}$: $A = \oint_{\mathcal{C}} x\,dy = \oint_{\mathcal{C}} -y\,dx = \frac{1}{2}\oint_{\mathcal{C}} x\,dy - y\,dx$
- Make sure that you understand and know how to apply ALL the formulas given in the cheat sheet!