Must Know Material for Midterm#3 - M252 - Calculus III - Fall 2022

List of the material that MUST be second nature to you in preparation for MT#3:

- **Chap. 15 Multiple Integrals:**
  15.4: Apps. of Double Integrals (only $M=\iint \rho$)
  15.5: Surface Area
  15.6: Triple Integrals
  15.7: Triple Integrals in Cylindrical Coordinates
  15.8: Triple Integrals in Spherical Coordinates
  15.9: Change of Variables in Multiple Integrals
- **Chap. 16: Vector Calculus:**
  16.1: Vector Fields
  16.2: Line Integrals
  16.3: The Fundamental Theorem for Line Integrals
  16.4: Green's Theorem

You must be very confident with the following basic and fundamental topics/formulas/techniques:

- Need to know the material about partial derivative from MT#2.
- If a constant is not defined (like an α or any other symbol), do NOT assign a value for it. It is a fixed scalar (or vector) and you should just leave it as it is.
- When dealing with functions of several variables, be careful about which variable you are doing the derivative (or integral) with respect to! [Remember the: "Integral (or derivative) with respect to what?"]
- Gradient: Nabla operator $= \nabla = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle$ ‖ gradient$(f) = \overrightarrow{\nabla f}(x,y,z) = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle f = \langle f_x, f_y, f_z \rangle$.
- Direction of steepest ASCENT is given by gradient $\nabla f(x,y,z)$ ‖ Gradient vector $\perp$ level curves.
- Polar: $\iint_{\mathcal{D}} f(x,y)\,dA = \iint_{\mathcal{D}} f\left(r\cos(\theta), r\sin(\theta)\right) \boxed{r}\,dr\,d\theta$. ‖ $r^2 = x^2+y^2$, $x = r\cos(\theta)$, $y = r\sin(\theta)$.
- Cylindrical: $\iiint_{\mathcal{B}} f(x,y,z)\,dV = \iiint_{\mathcal{B}} f(r\cos(\theta), r\sin(\theta), z) \boxed{r}\,dr\,d\theta\,dz$.
  with $r^2 = x^2+y^2+z^2$, $x = r\cos(\theta)$, $y = r\sin(\theta)$, $z = z$.
- Spherical: $\iiint_{\mathcal{B}} f(x,y,z)\,dV = \iiint_{\mathcal{B}} f\left(\rho\sin(\phi)\cos(\theta), \rho\sin(\phi)\sin(\theta), \rho\cos(\phi)\right) \boxed{\rho^2\sin(\phi)}\,d\rho\,d\theta\,d\phi$.
  with $\rho^2 = x^2+y^2+z^2$, $x = \rho\sin(\phi)\cos(\theta)$, $y = \rho\sin(\phi)\sin(\theta)$, $z = \rho\cos(\phi)$.
- Type I, type II, and type III for triple integrals.
- Surface area: $S = \iint_{\mathcal{D}} \sqrt{f_x^2+f_y^2+1}\,dA$.
- Volume: $V(\mathcal{B}) = \iiint_{\mathcal{B}} dV$
- Mass: $M(\mathcal{B}) = \iiint_{\mathcal{B}} \rho(x,y,z)\,dV$ where $\rho =$ density.
- Change of variables $(x,y,z) \to (u,v,w)$: $\iiint f(x,y,z)\,dV = \iiint f(u,v,w)\,|J|\,du\,dv\,dw$
  where $J = \left|\frac{\partial(x,y,z)}{\partial(u,v,w)}\right|$ is the Jacobian of the transformation. [Similar for double integrals]
- Line integrals for *scalar* fields:
  ◦ 2D: $\int_{\mathcal{C}} f(x,y)\,dS = \int_{t=a}^{b} f\left(x(t), y(t)\right)\sqrt{(dx/dt)^2+(dy/dt)^2}\,dt$ ‖ $x = x(t)$ & $y = y(t)$ for $a \le t \le b$
  ◦ 3D (and 2D): $\int_{\mathcal{C}} f(x,y,z)\,dS = \int_{t=a}^{b} f\left(\vec{\boldsymbol{r}}(t)\right)|\vec{\boldsymbol{r}}\,'(t)|\,dt$ ‖ $\vec{\boldsymbol{r}}(t) = \langle x(t), y(t), z(t)\rangle$ for $a \le t \le b$
- Line integrals for *vector* fields: $\int_{\mathcal{C}} \vec{\boldsymbol{F}}\cdot d\vec{\boldsymbol{r}} = \int_{\mathcal{C}} \vec{\boldsymbol{F}}\cdot\vec{\boldsymbol{T}}\,dS = \int_{t=a}^{b} \vec{\boldsymbol{F}}\cdot\vec{\boldsymbol{r}}\,'\,dt$
- Work: $\vec{\boldsymbol{F}}(x,y,z) = \langle P(x,y,z), Q(x,y,z), R(x,y,z)\rangle$: $W = \int_{\mathcal{C}} \vec{\boldsymbol{F}}\cdot d\vec{\boldsymbol{r}} = \int_{t=a}^{b} \vec{\boldsymbol{F}}\cdot\vec{\boldsymbol{r}}\,'\,dt = \int P\,dx + Q\,dy + R\,dz$
- Fundamental theorem: $\int_{\mathcal{C}} \overrightarrow{\nabla f}\cdot d\vec{\boldsymbol{r}} = f\left(\vec{\boldsymbol{r}}(b)\right) - f\left(\vec{\boldsymbol{r}}(a)\right)$
- A vector field is conservative if it comes from the gradient of a scalar field.
- $\vec{\boldsymbol{F}}(x,y) = \langle P(x,y), Q(x,y)\rangle$ conservative $\Leftrightarrow P_y = Q_x$ [if on a simply-connected domain].
- Green's theorem (GT): $\oint_{\mathcal{C}} P\,dx + Q\,dy = \iint_{\mathcal{D}} (Q_x - P_y)\,dA$ [path must be oriented *anti-clockwise*].
- Punching holes: *inner* paths must be oriented *clockwise* [i.e., *opposite*]. Remember: $\oint_{-\mathcal{C}} = -\oint_{\mathcal{C}}$
- Area using GT: Area enclosed by $\mathcal{C}$: $A = \oint_{\mathcal{C}} x\,dy = \oint_{\mathcal{C}} -y\,dx = \frac{1}{2}\oint_{\mathcal{C}} x\,dy - y\,dx$
- Make sure that you understand and know how to apply ALL the formulas given in the cheat sheet!