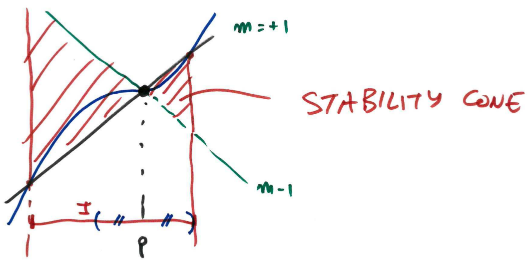
5.1



$I \subset B(p)$

5.2

1.6 The logistic map $G(x) = 4x(1-x)$

$a=4$

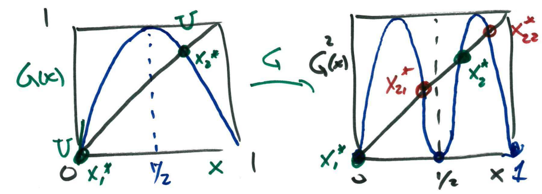
$g_a(x) = ax(1-x)$

$a = \underset{p=1}{2},\ \underset{p=2}{3.3},\ \underset{p=4}{3.5}\ \dots\ \boxed{a=4}$?

Period-1 : $G(x) = x$

$4x(1-x) = x \Rightarrow \begin{cases} x_1^* = 0 \\ x_2^* = 3/4 \end{cases}$

Stab : check ~~it is~~ they are U :

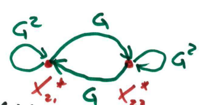


5.3

x_{21}^* & x_{22}^* are a p-2 orbit:

$G(x_{21}^*) = x_{22}^*$

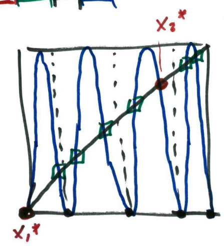
$G(x_{22}^*) = x_{21}^*$



Period 3 :

$G^3(x)$

$G^3(x)$



8 pts of G^3 − 2 pts of G

= 6 pts

=

2 × period-3 orbits

5.4

Period n

- From graph → 2^n fixed pts for $G^n(x)$

⚠ we need to substract f. pts comming from lower period to find TRUE period-n orbits !!!

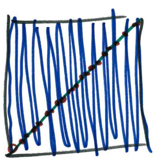
period n	# of fixed pts G^n	# of f. pts due to lower periods	# of TRUE period n-pts	# of period-n orbits
1	$2^1 = 2$	0	$2-0=2$	$\frac{2}{1} = 2$
2	$2^2 = 4$	2	$4-2=2$	$\frac{2}{2} = 1$
3	$2^3 = 8$	2	$8-2=6$	$\frac{6}{3} = 2$
4	$2^4 = 16$	$2\times1 + 1\times2 = 4$	$16-4=12$	$\frac{12}{4} = 3$
5	$2^5 = 32$	2×1	$32-2=30$	$\frac{30}{5} = 6$
6	$2^6 = 64$	$2\times1 + 1\times2 + 2\times3$	$64-10=54$	$\frac{54}{6} = 9$

5.5

- If $n = \text{prime} > 1 \Rightarrow$ there are
  - $\frac{2^n-2}{n}$ periodic orbits of period n
  - we just need $2^n - 2 > n$

$\boxed{?}$ HW: Do orbits of ANY period always exist? A: YES !!!

Q: what about stability ?



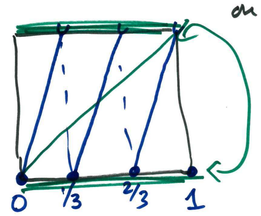
A: ALL UNSTABLE !!!

5.6

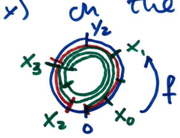
1.7 Sensitive dependence to ICs

Ex: 1.9 Consider $f(x) = 3x \pmod 1$ on $[0, 1]$

f(x)



Consider $f(x)$ on the ⊙ of circumference = 1

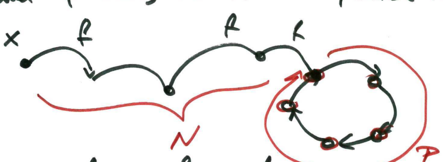


5.7

Def: x is eventually periodic with period P for f if for some $N > 0$

$f^{n+P}(x) = f^n(x)$ for $\forall\, n > N$

and P being the smallest possible integer.



Ex: $x = 1/3 \xrightarrow{f} 0 \xrightarrow{f} 0 \xrightarrow{f} 0 \dots$

Def 1.10: let f be a map on $\mathbb{R}$. A pt x_0 has SENSITIVE DEPENDENCE ON ICs if there is a non-zero distance d such that some pts arbitrarily

5.8

close to x_0 are eventually mapped at least d units from the corresponding image of x_0.

$\exists\, d, k > 0$ s.t. $\forall N_\varepsilon(x_0)\ \exists\, x \in N_\varepsilon(x_0)$

s.t. $|f^k(x) - f^k(x_0)| \geq d$