Chap 2: 2D maps

1D : $x_{n+1} = f(x_n)$ $\quad x_n \in \mathbb{R}$, $f(x_n) \in \mathbb{R}$

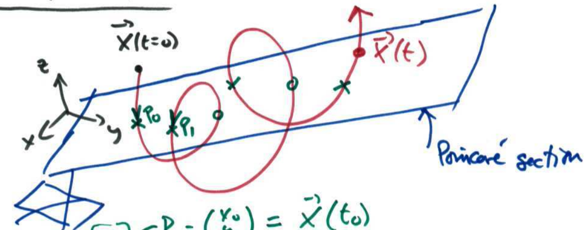
2D : $\vec{x}_{n+1} = F(\vec{x}_n)$ $\quad \vec{x}_n = \begin{pmatrix} x_n \\ y_n \end{pmatrix}$

$F = \begin{pmatrix} F_x \\ F_y \end{pmatrix}$

$\begin{pmatrix} x_n \\ y_n \end{pmatrix} \xmapsto{F} \begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix}$

$$x_{n+1} = F_x(x_n, y_n) = f(x,y)$$
$$y_{n+1} = F_y(x_n, y_n) = g(x,y)$$

Poincaré sections



F: $p_0 = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \vec{x}(t_0)$

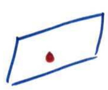
F: $p_1 = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \vec{x}(t_1)$

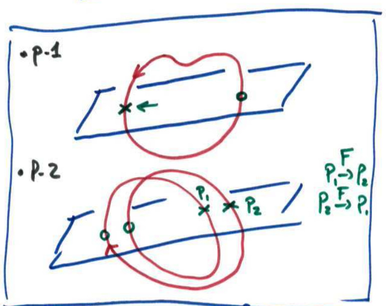
p_2 ⋮

2D map.

Periodic orbits

- F. pt.





Poincaré * 3D cont. flow $\longrightarrow$ 2D discrete Map.

* (N+1)D cont $\longrightarrow$ ND discrete

Henon Map:

$$\begin{cases} x_{n+1} = a - x_n^2 + b\,y_n \\ y_{n+1} = x_n \end{cases} \quad \text{// 2D Map}$$

$\Rightarrow$ $y_{n+2} = x_{n+1} = a - x_n^2 + b\,y_n$

$\Rightarrow$ $\boxed{y_{n+2} = a - y_{n+1}^2 + b\,y_n}$ 2d order recurrence

2.2 Stability: sinks, sources & saddle

Def 2.2: let f be a map on $\mathbb{R}^m$ and let p be a f.pt. ($p = f(p)$)

- If there exists $\varepsilon > 0$ such that any $v \in N_\varepsilon(p)$, $v \neq p$, we have $f^k(v) \xrightarrow{k \to \infty} p$ then p is SINK (S')
- If - - - - - - - - - - - - for some $k \geq 0$: $f^k(v) \notin N_\varepsilon(p)$ then p is a SOURCE (U)

Fig: SOURCE.

SINK: