Ex: Middle-third Cantor set:

- start with $x_0 \in [0,1]$
- flip coin
  * heads: move 2/3 of the way towards 1
  * tails: — " — " — " — 0
- repeat

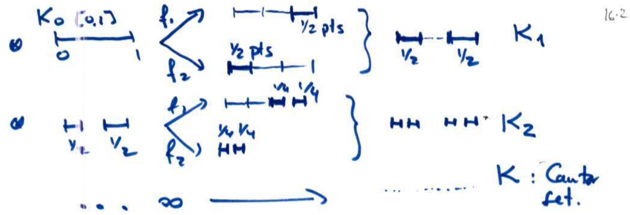
Q: where does x_0 go to?

A: $x_0 \to K$

Iterated Function Systems (IFS)

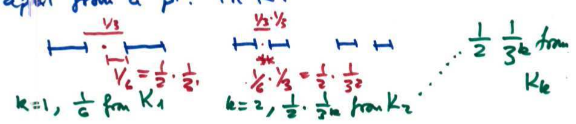
IC: $x_0 \in [0,1]$: $\begin{cases} f_1(x) = \frac{2}{3} + \frac{x}{3} \\ f_2(x) = \frac{x}{3} \end{cases}$

Q: where is x_0 mapped?



+ Any IC $\to$ K
+ If $x_0 \in K \to x_i \in K \; \forall i$
+ If $x_0 \notin K \to \{x_i\}_{i=1}^{\infty}$ tends to K asymptotically.

- At iterate k: you'll be @ most $\frac{1}{6} \cdot \frac{1}{3^k}$ apart from a pt. in K:



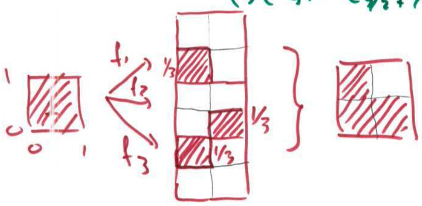
Def 4.4: An IFS on $\mathbb{R}^m$ is a collection of r maps: $\{f_1, \dots, f_r\}$ on $\mathbb{R}^m$ together with their respective probabilities $\{P_1, \dots, P_r\}$. $[\sum P_i = 1]$

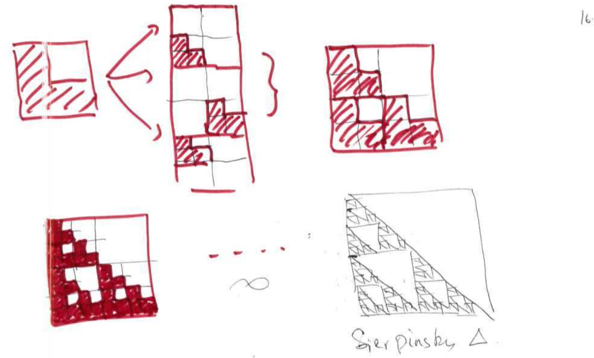
Ex: Sierpinski triangle (2D)

IFS: $f_1\binom{x}{y} = \binom{x/2}{y/2}$, $P_1 = 1/3$

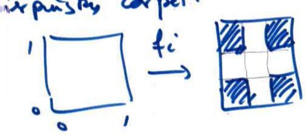
$f_2\binom{x}{y} = \binom{x/2 + 1/2}{y/2}$, $P_2 = 1/3$

$f_3\binom{x}{y} = \binom{x/2}{y/2 + 1/2}$, $P_3 = 1/3$

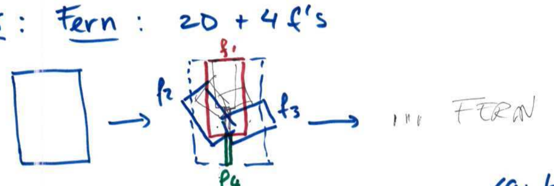




Ex: Sierpinsky carpet.



Ex: Fern: 2D + 4 f's

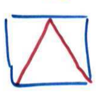
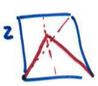


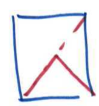
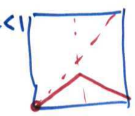
$f_i\binom{x}{y} = M_i\binom{x}{y} + V_i$ $\quad M_i = \begin{pmatrix} a_i & b_i \\ c_i & d_i \end{pmatrix}$ $\quad V_i = \binom{e_i}{f_i}$

4.3 Fractals from deterministic systems

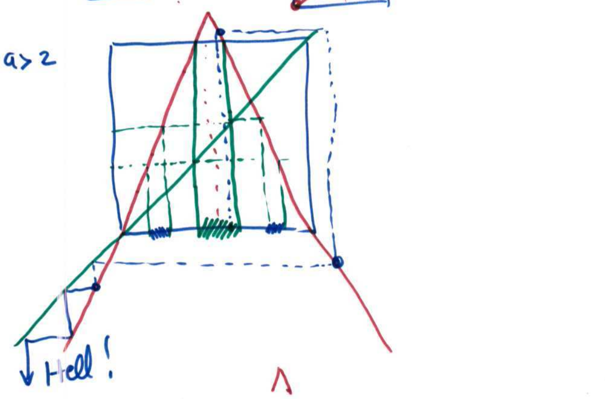
Tent map:

$$T_a(x) = \begin{cases} ax & x \le 1/2 \\ a(1-x) & x > 1/2 \end{cases}$$

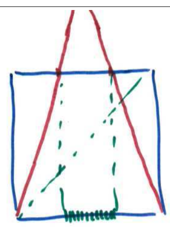
$a = 2$:  $1 < a < 2$ 

$a = 1$  $0 < a < 1$ 

$a > 2$



$T_3(x)$:



∴ * All ICs that $\in$ K will never leave $[0,1]$

* All ICs $x_0 \in \underbrace{[0,1] \setminus K}_{\text{Complement of K}}$ will go to Hell!

⚠ If $x_0 \in K \Rightarrow \{x_0, x_1, \dots\}$ is a chaotic orbit with $\lambda = \ln 3$!!!