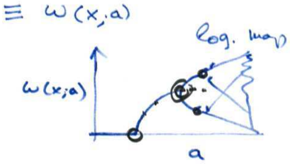
Chap 11 : Bifurcations

Bif. diag. $\equiv \omega(x;a)$



Def: A bif. pt. is a value for the parameter of a map where the existence and/or stability of a f.pt / period-orbit / chaotic orbit changes.

(A) 1D maps } area contrac.
(B) 2D maps } area preserv.

(A) 1D Bifurcations

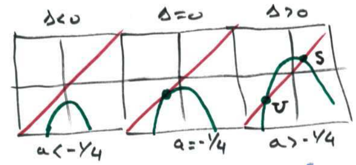
(A1) Saddle-node Bif

Ex: $f(x) = a - x^2$

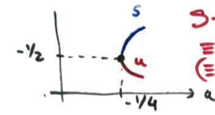
- f.pts: $a - x^2 = x \Rightarrow x^2 + x - a = 0$
  $\Rightarrow x_{1/2} = \frac{-1 \pm \sqrt{1+4a}}{2}$

$\Delta = 1 + 4a$: $\begin{cases} \Delta > 0 \to x_1 \,\&\, x_2 \\ \Delta = 0 \to x_1 = x_2 \\ \Delta < 0 \to \text{No f. pt.} \end{cases}$

$\Delta > 0 \Rightarrow 1 + 4a > 0 \Rightarrow a > -1/4$



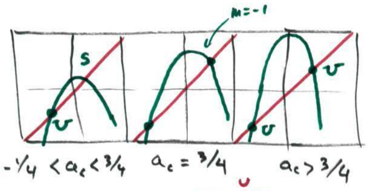
Bif. dig:

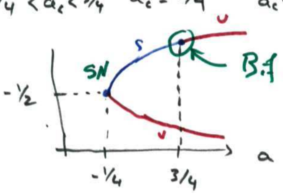


S-N bif.
≡ blue sky bif.
(≡ tangent bif.)

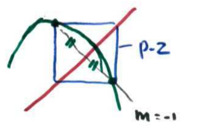
(A2) Period-doubling bif

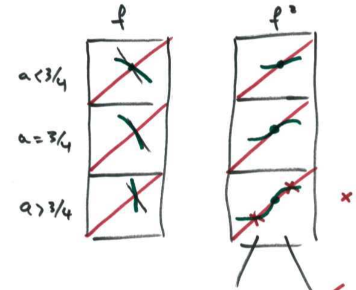
Same map $f(x) = a - x^2$ but ↑a further



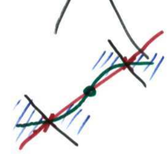


Q: what happens @ $a = 3/4$?
$a = 3/4 + \varepsilon$

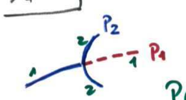




x, x = P-2 orbit.

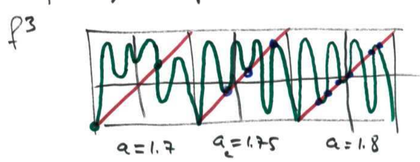


Bif. dig

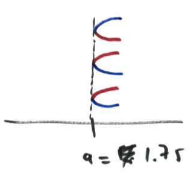


Period-doubling bif.

Ex: SN for logistic map



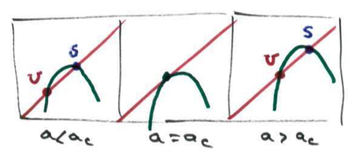
Bif. diag



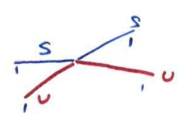
SN for P-3.

(A3) Transcritical Bif.

Scenario:



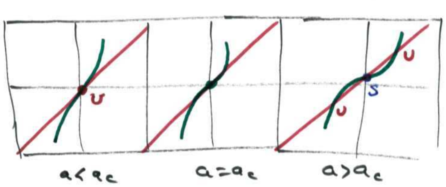
bif. diag:



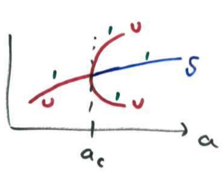
Transc. Bif.
→ collision
&
exchange of stability

Not typical because more conditions necessary than SN & Period-doub.

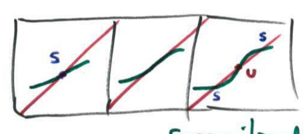
(A4) Pitchfork

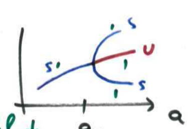


Bif. diag:



Subcritical Pitchfork bif.



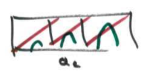


Supercritical Pitchfork. a_c

Detecting bifurcations:

$f(x_c; a_c) = x_c$

(A1) SN



① $\left.\frac{df}{dx}\right|_{(x_c, a_c)} = 1$

② $\left.\frac{d^2f}{dx^2}\right|_{(x_c, a_c)} \neq 0$] No I.P.

③ $\left.\frac{df}{da}\right|_{(x_c, a_c)} \neq 0$

25:00

A2 Period doubling.

- $\left.\frac{df}{dx}\right|_{(x_c,a_c)} = -1$
- $\left.\frac{d^2f}{dx^2}\right|_{(x_c,a_c)} \neq 0$] No. I.P
- $\left.\frac{df}{da}\right|_{(x_c,a_c)} \neq 0$

A3 Transcritical

- $\left.\frac{df}{dx}\right|_{(x_c,a_c)} = +1$
- $\left.\frac{d^2f}{dx^2}\right|_{(x_c,a_c)} \neq 0$
- $\left.\frac{df}{da}\right|_{(x_c,a_c)} = 0$

A4 Pitchfork

$\left.\frac{df}{da}\right|_{(x_c,a_c)} = 0$

- $\left.\frac{df}{dx}\right|_{(x_c,a_c)} = 1$
- $\left.\frac{d^2f}{dx^2}\right|_{(x_c,a_c)} = 0$
- $\left.\frac{d^3f}{dx^3}\right|_{(x_c,a_c)} \neq 0$ → >0 Supt, <0 Sub.