Ex: Extract ALL bifs of logistic map.

- $g_a(x) = ax(1-x) = ax - ax^2$
- $\frac{d}{dx} g_a(x) = a(1-2x)$
- $\frac{d}{da} g_a(x) = x(1-x)$

① $\boxed{\text{S.N.}}$

$\frac{dg}{dx} = 1$ 1c ⇒ ; $\frac{dg}{da} \neq 0$ 1b ⇒ ; $g_a(x) = x$ 1a ⇒

$x_c(1-x_c) \neq 0$ 1b

$\boxed{a_c(1-2x_c) = 1}$ 1c

$a_c x_c(1-x_c) = x_c$ 1a

1a: $\begin{cases} x_c = 0 \\ \boxed{a_c(1-x_c) = 1} \end{cases} \overset{a_c \neq 0}{\Longrightarrow}$

$\boxed{x_c = 0}$ ⇒ 1b Not satisfied

$\boxed{x_c \neq 0}$ $\begin{cases} a_c(1-x_c) = 1 \\ a_c(1-2x_c) = 1 \end{cases}$ $a_c(1-x_c) = a_c(1-2x_c)$

$a_c \neq 0 \Rightarrow 1-x_c = 1-2x_c$

$\Rightarrow x_c \neq 0$

∴ $\boxed{\text{No. S-N bif}}$

② Period-doubling:

- $g_a(x_c) = x_c \Rightarrow a x_c(1-x_c) = x_c$ (2a)
- $\frac{dg}{dx} = -1 \Rightarrow a_c(1-2x_c) = -1$ (2b)
- $\frac{dg}{da} \neq 0 \Rightarrow x_c(1-x_c) \neq 0$ (2c)

(2a): $\boxed{x_c = 0}$ or $\begin{cases} a_c(1-x_c) = 1 \\ a_c(1-2x_c) = -1 \end{cases}$ $a_c(1-x_c) = 1 = -a_c(1-2x_c)$

& $\boxed{\cancel{a_c = -1}}$ $\quad a_c \neq 0 \Rightarrow (1-x_c) = -(1-2x_c)$

$0 < a \leq 4$

$\Rightarrow 1 - x_c = -1 + 2x_c$

$\Rightarrow 2 = 3x_c \Rightarrow \boxed{x_c = 2/3}$

$\overset{2b}{\Longrightarrow} a_c(1 - 2 \cdot 2/3) = -1$

$\Rightarrow a_c(1 - 4/3) = -1$

$\Rightarrow a_c(-\frac{1}{3}) = -1$

$\Rightarrow \boxed{a_c = 3}$

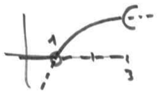
- $\frac{d^2 g}{dx^2} \neq 0$

$= \frac{d}{dx}(a(1-2x))$

$= -2a_c \overset{a_c=3}{\Longrightarrow} -6 \neq 0$

∴ @ $(x_c, a_c) = (2/3, 3)$ there is a period-doubling bif. !!!

③ transcritical:



$(a_c, x_c) = (1, 0)$

$\frac{dg}{dx} = a_c(1-2x_c) = 1(1-2\cdot 0) = 1$ ✓

$\frac{dg}{da} = x_c(1-x_c) = 0$ ✓

$\frac{d^2g}{dx^2} = -2a_c = -2 \neq 0$ ✓

∴ there is ONE TRANSCRIT. B.f. @ $(a_c, x_c) = (1, 0)$

④ Pitchfork: .... No way to satisfy eqns ⇒ No Pitchfork for log. map.

Genericity of bifs:

Ex ① SN: • $f(x) = a - x^2 \xrightarrow{\text{pert.}} \underbrace{g(x)}_{\text{perturbed map}} = f(x) + b$

- $g(x) = a - x^2 + b$

$= (a+b) - x^2 = \overset{a+b}{A} - x^2$

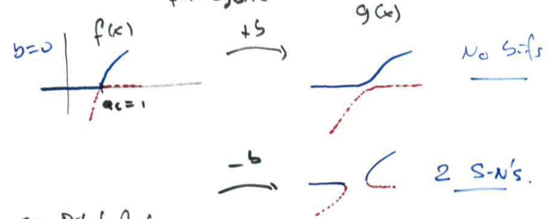
$\Rightarrow A_c = -1/4 \Rightarrow a_c = A_c - b = -1/4 - b$

∴ SN IS GENERIC !!!

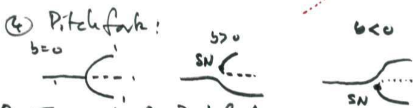
Ex ② period doubling → GENERIC

Ex ③ transcritical

$g(x) = \underbrace{ax(1-x)}_{f(x) = \text{logistic}} + b$



Ex: ④ Pitchfork:



∴ Transcrit. & Pitchfork → NOT GENERIC.