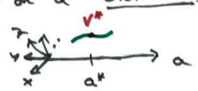
Project presentations:

Wed Dec 12^{th} 1:00 pm – 4:00 pm (Here)

11.3 continuability:

Q: when does a f. pt. retain its properties (existence + stability) if we change param. values?

Def: 11.6 Let f_a be a 1-param family of maps on $\mathbb{R}^n$. We say that a f. pt. v^* of f_{a^*} is locally continuable, or simply continuable, if the f. pt. of f_a near a^* lies on a continuous path.

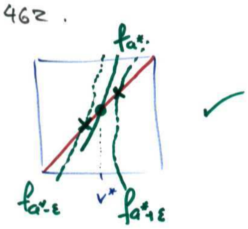


Theo 11.7: * 1D a) f_a on $\mathbb{R}^1$, if $f'_{a^*}(v^*) \neq 1$ $\Rightarrow$ (a^*, v^*) is continuable.

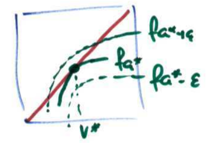
* ND b) f_a on $\mathbb{R}^n$, if Eigs$(Df_{a^*}(v^*)) \neq 1$ $\Rightarrow$ (a^*, v^*) is continuable

Proof: textbook p. 462.

geom. proof * $\boxed{1D}$



* $f'_{a^*}(v^*) = 1$



11.4 : B.f. in 1D maps ✓

11.5 B.f. of 2D maps: area contracting case

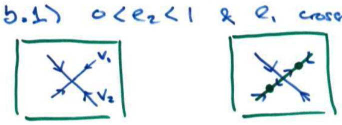
* $f_a(x,y)$ such that on the region of interest one eval e_2 is always $|e_2|>1$ or always $|e_2|<1$
  $\Rightarrow$ all possible bifs are the same as in 1D.
* area contracting $|\mathrm{Det}(J)|<1$
  * $e_1 \cdot e_2 = \mathrm{Det}(J) \Rightarrow |e_1 e_2| < 1$

a) if $|e_2|>1 \Rightarrow |e_1|<1 \Rightarrow$ No. bif :(

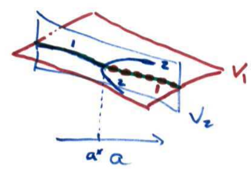
b) if $|e_2|<1 \Rightarrow |e_1|$ → $|e_1|<1$, $|e_1|=1$ ← possible bif :), $|e_1|>1$

b) Since $|e_2|<1$ near the bif. the evec of e_2

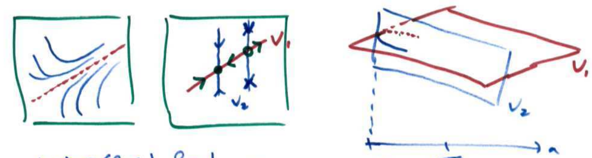
Cases: b.1) $0<e_2<1$ & e_1 crossing -1



→ Period doubling



b.2) $0<e_2<1$ $e_1=1$: S-N



b.3) $0<e_2<1$, $e_1=1$, ... Pitchfork.

b.4) $0<e_2<1$, $e_1=1$ transcritical.

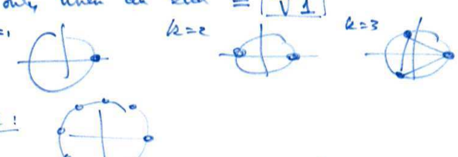
11.6 B.f. in 2D maps: area-preserving case

* in 2D (ND) evals might be complex.
* detect bifs' : $|e|=1$

Result: (Ex T11.10) Let f be a map on $\mathbb{R}^n$ and $\vec{p}$ a fixed pt.

a) $a+ib$ is eval $\Rightarrow$ $\overline{a+ib} = a-ib$ is also an eval.

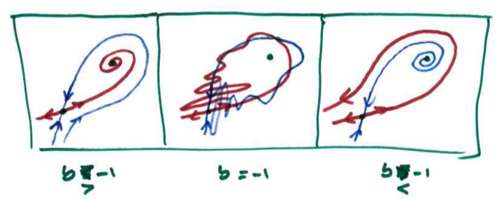
b) the bif. of a period-k orbit happens only when an eval $= \boxed{\sqrt[k]{1}}$



Ex: 11.14 $H_b: \begin{cases} -3/4 - x^2 + by \\ x \end{cases}$ Hénon : $a=-3/4$, b varies.

$DH_b = \begin{pmatrix} -2x & b \\ 1 & 0 \end{pmatrix}$

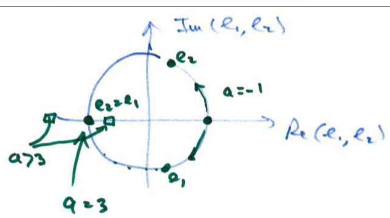
$\det(DH_b) = b$ → $|b|<1$: area contract. ; $|b|=1$: area preserving ; $|b|>1$: area expanding



Example: 11.15. $H_a(x,y) = \begin{cases} a - x^2 - y \\ x \end{cases}$ $b=-1$

$J = \begin{pmatrix} -2x & -1 \\ 1 & 0 \end{pmatrix}$, $|J| = 1$, $|e_1 e_2| = 1$

* F. pts : 2 f. pts: $x_\pm = y_\pm$
* Stab: 2 complex conj. evals $e_2 = \bar{e}_1$ + $|e_1 e_2| = 1$
  $\Rightarrow |e_1 \bar{e}_1| = 1$
  $\Rightarrow |e_1||\bar{e}_1| = 1 \Rightarrow |e_1| = 1 = |e_2|$



$a<-1$ No F. pts
$a=-1$ 1 f. pt
$-1<a<3$