AI for DS stripe [following our AI paper]: 3/9/06

$$H = \iint \left[\frac{|u_x|^2}{2} + \frac{|u_y|^2}{2} + \frac{1}{2}(|u|^2-\mu)^2 + V|u|^2 \right] dx\,dy$$

$1-t^2 = s^2$

$$u = e^{-i\mu t}\left[\sqrt{\mu-v^2}\tanh\left(\sqrt{\mu-v^2}(x-x_0)\right) + iv\right]$$

$t^2 = 1-s^2$

- $|u|^2-\mu = (\mu-v^2)\left[\tanh^2\right] + v^2 - \mu = (\mu-v^2)(1-\mathrm{sech}^2) + v^2 - \mu = \cancel{\mu}-\cancel{v^2}-(\mu-v^2)s^2+\cancel{v^2}-\cancel{\mu} = -(\mu-v^2)s^2 = \ldots\,\mathrm{sech}^2\left(\sqrt{\mu-v^2}(x-x_0)\right)$
- $|u_x|^2 = \left|\sqrt{\mu-v^2}\tanh'(\quad)\right|^2 = \left[(\mu-v^2)\,\mathrm{sech}^2(\ )\right]^2 = (\mu-v^2)^2\,\mathrm{sech}^4$

1D

$$\Rightarrow H_{1D} = \int \left(\frac{1}{2}(\mu-v^2)^2\,\mathrm{sech}^4 + \frac{1}{2}(\mu-v^2)^2\,\mathrm{sech}^4\right)dx$$

$$\Rightarrow H_{1D} = \int (\mu-v^2)^2\,\mathrm{sech}^4 = (\mu-v^2)^2\cdot\frac{4}{3\sqrt{\mu-v^2}} = \frac{4}{3}(\mu-v^2)^{\overbrace{2-1/2}^{3/2}}$$

$$\Rightarrow \boxed{H_{1D} = \frac{4}{3}(\mu-v^2)^{3/2}} \checkmark$$

$u = f(x-X)\,e^{ig(x-X)}$, $X = X(y)$
- $|u_y| = |-X_y f'e - iX_y g' f e| = |X_y|\,|f'e + ig'fe|$
- $|u_x| = |f'e + ig'fe|$

$\Rightarrow |u_y| = |X_y|\,|u_x|$

Landau dyn: $\mu \to \mu - V(x)$

$$\Rightarrow H_{1D} = \frac{4}{3}\left[\mu - V - v^2\right]^{3/2} = \text{const}$$

$$\overset{d/dt}{\Rightarrow} \frac{d}{dt}\left[\mu-V-v^2\right]^{3/2} = 0 \Rightarrow \frac{d}{dt}\left[-V-v^2\right]\frac{3}{2}\left[\mu-V-v^2\right]^{1/2} = 0$$

$$\Rightarrow -\dot{x}_0 V'(x_0) - 2\ddot{x}_0\dot{x}_0 = 0 \Rightarrow \boxed{\ddot{x}_0 = -\frac{1}{2}V'(x_0)} \checkmark$$

2D

$$H_{2D} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\frac{1}{2}\left(|u_x|^2 + |u_y|^2\right) + \frac{1}{2}(|u|^2-\mu)^2\right]dx\,dy$$

$X_0 = X_0(y,t)$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\frac{1}{2}|u_x|^2(1+X_{0y}^2) + \frac{1}{2}(|u|^2-\mu)^2\right]dx\,dy$$

$$= \int_{-\infty}^{\infty}\left[(1+X_{0y}^2)\underbrace{\frac{1}{2}\int_{-\infty}^{\infty}|u_x|^2\,dx}_{\frac{2}{3}(\mu-v^2)^{3/2}} + \underbrace{\frac{1}{2}\int_{-\infty}^{\infty}(|u|^2-\mu)^2\,dx}_{\frac{2}{3}(\mu-v^2)^{3/2}}\right]dy$$

$$\Rightarrow \quad H_{2D} = \int_{-\infty}^{\infty} \left[\frac{2}{3} (\mu - v^2)^{3/2} \, (1 + X_{0y}^2 + 1) \right] dy$$

$$= \cancel{\int_{-\infty}^{\infty} \frac{4}{3} \int_{-\infty}^{\infty}} \quad = \int_{-\infty}^{\infty} \left[\frac{4}{3} (\mu - v^2)^{3/2} \left(1 + \frac{X_{0y}^2}{2}\right) \right] dy$$

$$\cancel{= \frac{4}{3} (\mu - v^2)^{3/2} \int_{-\infty}^{\infty}}$$

but $H_{2D} = \text{const} \quad \Rightarrow \quad \frac{d H_{2D}}{dt} = 0 \quad \Rightarrow \quad \frac{d}{dt} \left[\frac{4}{3} (\mu - v^2)^{3/2} \left(1 + \frac{X_{0y}^2}{2}\right) \right] = 0$

Landau dyn : $\mu \to \mu - V(x) \quad \Rightarrow \quad \boxed{E = \frac{4}{3} \int_{-\infty}^{\infty} \left(1 + \frac{X_{0y}^2}{2}\right) \left(\mu - V(X_0) - X_{0t}^2\right)^{3/2} dy}$

Eq. (6) in AI paper

$$\Rightarrow \frac{d}{dt} \left[\frac{4}{3} \left(\mu - V(X_0) - v^2\right)^{3/2} \left(1 + \frac{X_{0y}^2}{2}\right) \right] = 0$$

$v^2 = \dot{X}_0^2$

$$\Rightarrow \quad \cancel{-\dot{x}_0 V'(x_0) \tfrac{3}{2} 2 (\mu} \left(-\dot{X}_0 V'(X_0) - 2 \ddot{X}_0 \dot{X}_0\right) \left(1 + \frac{X_{0y}^2}{2}\right)$$

$$+ \left(\mu - V(X_0) - v^2\right)^{3/2} \; 2 X_{0yt} X_{0y} = 0$$

let us repeat using PGK notes from 3/Apr/17 → [PGK_notes_1I_new.jpg] 6/95d

$\boxed{\text{DS}}$ • $H_{2D} = \frac{1}{2}\iint_{-\infty}^{\infty} \left[|u_x|^2 + |u_y|^2 + (|u|^2 - \mu)^2 \right] dx\,dy$

• $u = e^{-i\mu t}\left[\sqrt{\mu - v^2}\tanh\left(\sqrt{\mu - v^2}\,(x - x_0)\right) + i v\right]$

$\Rightarrow |u_x|^2 = (\mu - v^2)^2 \operatorname{sech}^4$, $\quad |u_y|^2 = x_0^2\,|u_x|^2$

$\Rightarrow \int |u_x|^2 = (\mu - v^2)^2 \frac{4}{3}\frac{1}{\sqrt{\mu - v^2}}$

• $|u|^2 - \mu = -(\mu - v^2)\operatorname{sech}^2 \xrightarrow{\int(\,)^2} + (\mu - v^2)^2 \frac{4}{3}\frac{1}{\sqrt{\mu - v^2}}$ $=$

see p. 3/

see p. 3/ & 4/

$\underline{V=0} \Rightarrow H_{2D} = \frac{4}{3}\int_{-\infty}^{\infty}\left(1 + \frac{X_{oy}^2}{2}\right)\left(\mu - X_{ot}^2\right)^{3/2} dy$

but $\frac{dH_{2D}}{dt} = 0 \overset{\div 4/3}{\Rightarrow} \int_{-\infty}^{\infty}\Big[\big(\underbrace{\tfrac{2}{2} X_{oyt}}_{dv} X_{oy}\big)\underbrace{(\mu - X_{ot}^2)^{3/2}}_{u} + \left(1 + \frac{X_{oy}^2}{2}\right)\frac{3}{2}(-2X_{ott}X_{ot})(\mu - X_{ot}^2)^{1/2}\Big] dy = 0$

$\int_{-\infty}^{\infty} \underbrace{X_{oyt}}_{dv}\,\underbrace{X_{oy}\,(\mu - X_{ot}^2)^{3/2}}_{u}\,dy$ parts $\begin{cases} u = X_{oy}(\mu - X_{ot}^2)^{3/2} \to \frac{du}{dy} = \frac{d}{dy}[\;] \\ v = X_{ot} \leftarrow \frac{dv}{dy} = X_{oyt} \end{cases}$

$= \left[X_{ot}X_{oy}\cancel{(\mu - X_{ot}^2)^{3/2}} \right]_{-\infty}^{\infty} - \int X_{ot}\frac{d}{dy}\left[X_{oy}(\mu - X_{ot}^2)^{3/2}\right]$ $\quad \to 0$

$= -\int X_{ot}\left[X_{oyy}(\mu - X_{ot}^2)^{3/2} + X_{oy}\frac{3}{\cancel{2}}(-\cancel{2}X_{oty}X_{ot})\,(\mu - X_{ot}^2)^{1/2}\right]$

$\therefore \frac{dH_{2D}}{dt} = 0 \Rightarrow -\int X_{ot}\left[X_{oyy}(\mu - X_{ot}^2)^{3/2} - X_{oy}\,3(X_{oty}X_{ot})(\mu - X_{ot}^2)^{1/2}\right] + 3\left(1 + \frac{X_{oy}^2}{2}\right)(-X_{ott}X_{ot}(\mu - X_{ot}^2)^{1/2})$

$\Rightarrow +\int X_{ot}(\mu - X_{ot}^2)^{1/2}\left\{-X_{oyy}(\mu - X_{ot}^2) + 3X_{oy}X_{oty}X_{ot} - 3\left(1 + \frac{X_{oy}^2}{2}\right)X_{ott}\right\} dy = 0$

$\overset{\div(-3)}{\Rightarrow} \frac{1}{3}X_{oyy}(\mu - X_{ot}^2) - X_{oy}X_{oty}X_{ot} + \left(1 + \frac{X_{oy}^2}{2}\right)X_{ott} = 0$

$\Rightarrow \boxed{X_{ott}\left(1 + \frac{X_{oy}^2}{2}\right) = -\frac{1}{3}X_{oyy}(\mu - X_{ot}^2) + X_{oy}X_{ot}X_{oty}}$

same eq. as in PGK's notes

linear: $X_{tt} = -\frac{\mu}{3}X_{yy}$

Implementation: $X_0 \to X$, $v = X_t$

$\begin{cases} X_t = v \\ v_t = (1 + X_y^2/2)^{-1}\left[-\frac{1}{3}X_{yy}(\mu - v^2) + X_y v v_y\right] \end{cases}$